

**Journal of Stress Analysis** Vol. 1, No. 1, Spring – Summer 2016



# Strain Analysis by Digital Shearography on an Aluminium Plate with a Geometric Defect under Thermal Loading

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## Article info

#### Abstract

Article history: Received 2016.05.05 Received in revised form 2016.08.22 Accepted 2016.09.14

Keywords: Strain measurement Phase calculation Digital shearography Geometric defect

#### measurement. In this article strain analysis on defected plate has been studied by the digital shearography method along with a new technique for phase map measurement. For this purpose, an optical set-up known as modified Michelson interferometry system with two diode lasers were developed which are used as coherent optical source. To create phase shifting a piezo-electrical ceramic was used to miniature displacements of interferometry. Phase shifting technique was used to measure phases and differences between the phases. The strain was measured using the opperimental method and compared

Digital shearography is a non-destructive and non-contact method for strain-

The strain was measured using the experimental method and compared with numerical analysis results. Behavior of two graphs of experimental measurement and numerical analysis was approximately the same.

# Nomenclature

$\gamma$	Modulus of interferometry	$\phi(xy)$	Relative phase angle before loading
I(xy)	Intensity of one point on the picture	$I_0$	Intensity of two sheared images before
( )		-	the deformation
CCD	Charged coupled device	$\Delta$	Relative phase shift
$\delta x$	Amount of shear in the $x$ direction	DSPSI	Digital speckle pattern shearing inter-
			ferometry

# 1. Introduction

Increasing demands for a quality and highly reliable product requires effective methods of measurement and testing which are non-destructive, full-field, and uncontactable. Optical methods seem to be ideal for these purposes. The newly developed shearography method, also known as Speckle Pattern Shearing Interferometry (SPSI) is a coherent optical method of measurement and testing which is similar to holographic interferometry. This method, due to its relatively low sensitivity to the environment, has several advantages for industrial applications. Unlike holographic interferometry,

This is the reason why shearography is increasingly known as an important measuring tool in the industry. Digital shearography, which is also known as 'Speckle Pattern Shearing Interferometry' (SPSI), is a coherent optical method along with digital image processing. The optical theory in digital shearography is the same as Speckle Pattern Shearing Interferometry, but from a technical perspective, digital shearography is considered as a digital process that eliminates the process concerning the images appearance. This matter helps

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which is used for measuring deformations, it measures gradient deformations. Thus, strain data can be directly measured.

Strain analysis by digital shearography on an aluminium plate with a geometric defect under thermal loading: 1-7



**Fig. 1.** Principle of digital shearography: a) Experimental set-up and b) The two images are sheared in x direction [4].

to hasten testing, thus the shearogram can be observed in real time. Digital shearography, by using the phaseshift technique, allows for the automatic and numerical calculation of shearogram [1], providing new possibilities for applying shearography in the industry.

A method of obtaining pure in-plane strain by applying shearography is using two linearly independent directions of illumination (usually having identical reverse angles). The shearogram for both directions of illumination can be obtained from using the phase-shift technique [2,3].

The result of decreasing the phase map of both Shearograms yield a fringe pattern, depicting the pure in-plane strain components. And the result of increasing the 2-phase map of both shearograms are the fringe patterns of the pure out-of-plane or tilt components. Therefore, the pure in-plane strain components can be obtained directly from displacement data, without numerical differentiation.

#### 2. Principles of Digital Sheargraphy

The experimental system for digital shearography is shown in Fig. 1a; the test specimen was illuminated, using a wide-range light beam.

The light reflected from the plate surface was focused on the image plate of a charge-coupled device (CCD) camera. Therefore, a couple of horizontally sheared images were obtained from the testing specimen on the image plate of CCD, with a very small shift in the angle of mirror No. 1 from the vertical state (Fig. 1b). By shearing the two images, the Speckle Interferometry pattern was produced. The intensity distribution I(xy) of Speckle Interferometry pattern is given according to the following relation [4]:

$$I(xy) = I_0 [1 + \gamma \cos \phi(xy)] \tag{1}$$

where  $I_0$  indicates the intensity of these two sheared images before the deformation of the specimen. When the specimen is going under deformation, the intensity distribution of Speckle pattern changes very low.

$$I'(xy) = I_0 [1 + \gamma \cos \phi'(xy)]$$
<sup>(2)</sup>

These two intensities were recorded with a CCD camera; the result of subtraction between the two pieces of information obtained from these two images provided the fringe pattern, in other words, "digital shearogram" describing the relative phase-shift of  $[\delta = \phi'(xy) - \phi(xy)]$  resulted from the deformation in the specimen, shown in real-time on the monitor screen [5].

It was indicated that the relative phase-shift  $\Delta$ , having relation with displacement derivatives instead of displacement itself, was linked to a shearing relation of shearography [6]. If the shearing is in the direction of x, then  $\Delta$  is obtained from the following relation:

$$\Delta_x = \left(\frac{\partial u}{\partial x}k_s e_x + \frac{\partial v}{\partial x}k_s e_y + \frac{\partial w}{\partial x}k_s e_z\right)\delta x \qquad (3)$$

And if the shearing is in the direction of y, Eq. (3) will be as follows:

$$\Delta_y = \left(\frac{\partial u}{\partial y}k_s e_x + \frac{\partial v}{\partial y}k_s e_y + \frac{\partial w}{\partial y}k_s e_z\right)\delta x \qquad (4)$$

where v, u and w are respectively components of the displacement vector in the directions of x, y and z.  $e_x, e_y$  and  $e_z$  are unit vectors in the directions of x, y and z.  $\delta x$  and  $\delta y$  values are shearing in the directions of x and y; and  $k_s$  the sensitivity vector.



Fig. 2. Fixture of tensional loading on the Aluminum plate with circular defect.

It was indicated [7] that the sensitivity vector is the bisect of the angle between the radiationbeam and reflection [Fig. 1a].

In total, there are three unknowns  $I_0, \gamma$ , and  $\varphi$  in the intensity distribution in Eq. (1). Therefore, at least three measurements are required to determine the relative phase angle; and for every recorded intensity,

$$\phi = \arctan \frac{\sqrt{3}(I_3 - I_2)}{2I_1 - I_2 - I_3}$$
$$\phi = \arctan \frac{(I_4 - I_2)}{(I_1 - I_3)}$$

Therefore, the distribution of the relative phase angle  $\varphi$  can be calculated from the recorded intensities [8].

When the specimen was modified, three (or four) new images of modified intensities were recorded, while, as in the first case, the phase shift was done with the same data. The phase distribution  $\varphi$  of shift-phase interference pattern was also obtained the same as  $\varphi$ .

one 120-degree shift (for three measurements) or a 90degree shift (for four measurements) of the phase was created for a Michelson shearography beam. To create fine displacements from a variable voltage, a piezoelectric was used to create phase shift. The analysis of three (or four) intensity patterns produced three (or four) equations.

### 3. Experimental Procedure

Given that the loading procedure would be thermal in transient form and made on behind of a defective aluminum plate, dimensioned  $60 \times 25 \times 0.4$ mm, being clamped on two sides and given that the laser wavelength would be 630nm and the shearing amount in the experiment, 6mm; in order to apply fine displacements, a spiral heating resistance wire element for thermal loading was used which resistivity was Strain analysis by digital shearography on an aluminium plate with a geometric defect under thermal loading: 1-7

 $1.09(\mu\Omega.m)$  with the wire diameter of 0.5mm, whose image and three-dimensional diagram of its fixture are shown in Fig. 2.

In this study, the amount of strain was calculated according to the experimental digital shearography method and was compared with the results obtained from the finite element method (FEM). Furthermore, in this experiment an innovative method was used to calculate the phase, making the calculation time one step shorter than the conventional method.

As described in the previous section, in the traditional method, at first, before loading the specimen, image is recorded three or four times while the primary phase is calculated from these images and then is applied to the specimen loading. Next, by using the phase-shift method, three or four other images are created. Then, using these images, the secondary phase is calculated and in the following stage the secondary and primary phase shifts are calculated. As seen in Fig. 3, this procedure was performed by two lasers having identical and reverse angles, and by using the obtained phase shift, as well as, Eqs. (3) and (4), the amount of inplane strain and the pure out-of-plane component was calculated [3].





Fig. 3. Illumination from two equal angles.

However, the method used in this article is as follows: First, a pre-loading intensity was recorded.

$$I_1 = 2I_0 \left[ 1 + y \cos \phi \right] \tag{7}$$

And once the loading were done, four images would be

recorded, using phase-shift method.

$$I_{11} = 2I_0 [1 + y \cos \phi]$$

$$I_{22} = 2I_0 [1 + y \cos(\phi + 90^\circ)]$$

$$I_{33} = 2I_0 [1 + y \cos(\phi + 180^\circ)]$$

$$I_{44} = 2I_0 [1 + y \cos(\phi + 270^\circ)]$$
(8)

And then, each one of the intensities obtained after the loading was deducted from the intensity obtained before the loading, thereby four new intensities were obtained.

$$I'_{11} = |I_{11} - I_1|$$

$$I'_{22} = |I_{22} - I_1|$$

$$I'_{33} = |I_{33} - I_1|$$

$$I'_{44} = |I_{44} - I_1|$$
(9)

And using these four new intensities and Eq. (6), a new phase was obtained.

$$\phi = \frac{I'_{44} - I'_{22}}{I'_{33} - I'_{11}} \tag{10}$$

The phase obtained from Eq. (10) was placed in Eq. (11), resulted from Eq. (3), in which the shearing was in the x direction.

$$\phi_{+\theta} = \frac{2\pi\delta x}{\lambda} \left[ \sin(+\theta) \frac{\partial u}{\partial x} + (1 + \cos(+\theta)) \frac{\partial w}{\partial x} \right] \quad (11)$$

The same procedures were carried out for the laser with  $(-\theta)$  angle, and thus, Eq. (12) was obtained.

$$\phi_{-\theta} = \frac{2\pi\delta x}{\lambda} \left[ \sin(-\theta) \frac{\partial u}{\partial x} + (1 + \cos(-\theta)) \frac{\partial w}{\partial x} \right] \quad (12)$$

#### 4. Data Analysis and Results Discussion

From the five images recorded before and after loading and using Eqs. (9) and (10), the step-by-step phase tensor for the recorded intensity data, using laser No 1 was obtained, according to Fig. 4a.



Fig. 4a. Final phase tensor obtained from the image taken with laser 1 (right), discontinuous phase.

And by making the step-by-step phase continuous, using a laser No 1, the continuous phase tensor for the recorded intensity data was obtained as shown in Fig. 4b.



Fig. 4b. Final phase tensor obtained from the images taken with laser No 1 (right), continuous phase.

For one of the columns of the continuous phase tensor, the two-dimensional graph of the phase in Fig. 4c was obtained.



Fig. 4c. The two dimensional phase for rows 100 to 900 and column 300 of continuous phase tensor of laser No 1 (right).

The same procedures were repeated for the left-side laser (in identical loading) and its obtained phase is shown in Fig. 5.



**Fig. 5a.** Final phase tensor obtained from the images taken with laser No. 2 (left), discontinuous phase.

And by changing the step-by-step phase into a con-

tinuous one, the continuous phase tensor for laser No 2 (left) was obtained according to Fig. 5b.



Fig. 5b. Final phase tensor obtained from the images taken with laser No 2 (left), continuous phase.

For one of the columns of the continuous phase tensor, the two-dimensional graph of the phase in Fig. 5c was obtained.



Fig. 5c. The two-dimensional phase for rows 100 to 900 and column 300 of continuous phase tensor of laser No 2 (left).

In Fig. 6, the phase resulted from the radiation of two lasers in reverse and identical angles and their addition and subtraction graph is drawn.



**Fig. 6.** The phase graph for both left and right lasers, their addition and subtraction.

Strain analysis by digital shearography on an aluminium plate with a geometric defect under thermal loading: 1-7 6

#### 4.1. Experimental Calculation of Pure In-plane Strain

By subtraction of two Eqs. (11) and (12) from each other, Eq. (13) was resulted; where, the pure in-plane strain is obtained as it follows:

$$\phi_1 = \phi_{+\theta} - \phi_{-\theta} = \frac{4\pi\delta x(\sin\theta)}{\lambda}\frac{\partial u}{\partial x}$$
(13)

By placing the shearing amount of  $(\delta x = 6 \text{mm})$ , the laser wavelength ( $\lambda = 630 \text{nm}$ ), the illumination angle ( $\theta \approx 16^{\circ}$ ) and the amount of phase obtained from Eq. (13), the pure in-plane strain was obtained, according to Fig. 7.



**Fig. 7.** Graph of the pure in-plane strain component  $(\partial u/\partial x)$ .

#### 4.2. Finite Element Analysis of the Pure Inplane Strain

Assuming, 70 Gpa, 0.3,  $23 \times 10^{-6} \frac{m}{m \times {}^{\circ}C'} 2.7g.cm^{-3}$  and 900 J/Kg.K respectively for the elastic modulus, Poisson's ratio, expansion, density and specific heat of the aluminum specimen. The element type in FEM analysis was C3D8T (coupled temperature-displacement) for meshing and the number of elements were 1953. The strain was calculated according to Fig. 8.



Fig. 8. Results of numerical analysis of the finite element.

### 4.2.1. Comparison of the Results Obtained From the Experimental and Numerical Analysis

In this experiment, measuring was done in the direction of the axis x of the specimen with dimensions of  $60 \times 25 \times 0.4$ mm. The region under study (the white line on the image) is shown in Fig. 9; the experimental strain was graphed in this region for column 300 and the rows 100 to 900 of the phase tensor which is about 12mm. Also, the results obtained from the analysis of the finite element is shown in Fig. 8.



Fig. 9. Position of investigation on the specimen under stress.

By comparison of results obtained from numerical and experimental analysis, it can be seen that the finite element analysis results also nearly confirm the experimental test results with marginal error. Fig. 10 has done a comparison between these two analyses.



Fig. 10. Comparison of results obtained from numerical and experimental analysis.

As can be seen the power in both measurements is  $10^{-8}$ .

#### 4.3. Error Analysis

Errors which can be effective in calculating the strain are as follows:

- Calculation error of laser beam angle  $(\pm 1^{\circ})$
- Calculation error of shearing amount  $(\pm 0.1 \text{mm})$
- Error resulted from calculating the phase  $(\pm 0.01 \text{rad})$

Any of these errors and their combination can be effective in the strain graph. For the combination of positive and negative errors, the strain graph has been drawn in Fig. 11. As can be seen, the relative error of the experimental measurement is about 20.1 percent.



Fig. 11. Effects of probable errors and relative error.

## 5. Conclusions and Discussion

In this study, non-contact optical shearography method was used, along with the phase-shift method, for measuring in-plane and out-of-plane strain components. Numerical analysis was used as a procedure for validation. To perform the tests, an aluminum specimen with dimensions of  $60 \times 25 \times 0.4$ mm was used which had a defect in the center. And to apply the thermal stress, a spiral heating resistance wire element which resistivity was  $1.09(\mu\Omega.m)$  and the wire diameter of 0.5mm was used. To apply the phase-shift method, an innovative phase calculating algorithm was used. And finally, the strain was measured using digital shearography method whose results in shearing direction are shown in the graph.

During the experimental measurement some errors affected the obtained results which by taking them into consideration, the value of the relative error was determined to be about 20.1 percent. The comparison of experimental analysis with numerical analysis indicated the identical behavior in both strain graphs obtained from experimental measurement and numerical analysis. The strain power in both analyses is identical and  $10^{-8}$ . These two analyses almost had identical results with less error.

## Acknowledgements

The researchers would like to express thanks to Ms. PegahAskari and Mr. Yousef Pourveis for their humble assistance in the arrangement of the shearography system in the optics laboratory, as well as, programming.

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