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Elasticity Solution for Static Analysis of Sandwich Structures with Sinusoidal Corrugated Cores

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Abstract

Metal sandwich panels are three-dimensional structures widely used in industries mainly due to two distinct properties: low density and high strength. Although significant efforts have been made at research into corrugated sandwich panels, analytical solutions are still very few. This work wishes to present accurate analytical results of static analysis of corrugated sandwich panels. In order to determine equivalent properties of corrugated core in the thickness direction, energy method is used in conjunction with homogenization approach. Based on three-dimensional theory of elasticity, partial differential equations are reduced to ordinary differential equations by using the Fourier series. Analytical solutions for the stress and displacement fields are derived by using the state-space method in the thickness direction.
A detailed parametric study was conducted involving the dependency of out-of-plane properties on the corrugation geometrical parameters. Moreover, effects of these variables on the stress and displacement fields are discussed.

Nomenclature

A	cross-section	A	Length
B	Width	C_{ij}	equivalent constants
C_{ij}	stiffness elastic constants	E	elastic modulus
G	shear modulus	h_c	height of core
h_f	facing thickness	h_t	total height of panel
I	moment of inertia	L	Length
M_0	external moment	M	bending moment
N	internal normal force	P	half period of core
Q	external distributed force	T	internal tangential force
t_c	thickness of core sheet	V	external vertical force
u, v, w	displacement components in the x, y, and z	$\bar{U}, \bar{V}, \bar{W}$	non-dimensional displacement components
x, y, z	Cartesian reference coordinates, where x is along the corrugation direction		
Greek symbols			
γ_{ij}	shear strain	δ_H	horizontal displacement
δ_V	vertical displacement	E	normal strain
H	non-dimensional thickness coordinate	ν_{zx}	Poisson's ration
σ_i	normal stress	τ_{ij}	shear stress
φ	angle of tangent line		

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Subscripts			
0	$z = 0$	n, m	half-wave numbers in the x and y directions
xyz	reference coordinates		

1. Introduction

It is well-known that metallic sandwich panels are widely used in many engineering fields due to their low density, high strength, and easy fabrication. Corrugated sandwich panel is a special type of sandwich panel which has corrugated metallic core with alternate ridges and grooves shaped. Due to their extremely anisotropic behavior, corrugated sandwich panels are increasingly used in many industrial applications, particularly in aerospace engineering.

There are several articles discussing equivalent properties of corrugated sandwich panels to predict their mechanical behavior. On the basis of periodic nature of corrugated cores, many authors have used homogenization methods to predict the behavior of corrugated cores. One of the early investigations was conducted by Libove and Hubka [1] showing that shear effects are not negligible and should be considered in equivalent orthotropic plates. Briassoulis [2] used FEM to calculate the extensional and flexural rigidity of sinusoidal corrugated plates. By considering Mindlin-Reissner plate theory, Chang et al. [3] presented an equivalent two-dimensional orthotropic thick plate for corrugated-core sandwich plates; The bending analysis of corrugated sandwich panels was performed and indicated that obtained results are in good agreement with previous experimental investigations. Not all of the works are dedicated to homogenization techniques for corrugated cores; in [4], the authors have used homogenization theory to present the effective properties of sandwich panels with corrugated cores. Kirchhoff-Love theory was used to solve basic cell problems. However, they acknowledged that it is necessary to use complicated theories like MindlinReissner models to provide better results for stiffer panels.

Xia et al. [5] assumed that corrugated panel can be approximated by an orthotropic classical Kirchhoff plate. They used equivalent force method to derive equivalent stiffness terms for any corrugation shapes. Bartolozzi et al. studied the acoustic behavior of corrugated cores with sinusoidal shape [6]. They used energy approach to obtain the main parameters of corrugated cores and showed that the results are in good agreement with detailed finite element simulations. Mohammadi et al. [7] proposed an equivalent model for trapezoidal corrugated cores. Zheng et al. [8] proposed an equivalent plate model for corrugated cores by using classical shell theory. They presented a complete set of effective in-plate stiffness. Park et al. [9] extended the homogenization model for corrugated composite cores and presented explicit expressions to calculate effective extensional and bending stiffness for them; It was shown that effective stiffness and the anisotropy are considerably affected by ply angles. An excellent survey of the research work on the composite corrugated structures can be found in the work done by Dayyani et al. [10]

To the knowledge of the author, more literature is dedicated to calculate the equivalent in-plane properties of corrugated sandwich panels despite the fact that corrugated sandwich panels are three-dimensional structures and as a result of those extreme orthotropic natures, equivalent out-of-plane properties cannot be ignored.

In the present work, an attempt was made to determine the equivalent out-of-plane properties of corrugated sandwich panels which have not been considered yet.

Moreover, analytical solution for bending problem of corrugated sandwich panels was developed by incorporating both equivalent in-plane and out-of-plane properties into the three-dimensional theory of elasticity. First, partial differential equations were reduced to the ordinary differential equations by expanding the field variables to double Fourier series along in-plane directions. Then, coupled state-space equations were derived and solved by imposing the boundary conditions in the thickness direction.

2. Equivalent Orthotropic Properties in the z-Direction

A corrugated sandwich panel with length a, width b and total thickness h_t, as shown in Fig. 1, is considered. The corrugated sandwich panel is considered as a multi-layer composite plate consisted of two isotropic facing sheets with thickness h_f and one thick orthotropic plate having equivalent elastic constants instead of a corrugated core.

The objective of this section is to obtain the main equivalent out-of-plane parameters (i.e. elastic modulus, E_z and Poissons ratio, ν_{zx}). Note that for three-dimensional bending analyses, the out-of-plane properties of corrugated core are required which cannot be found in the previous investigations. Without loss of generality, sinusoidal shape was considered for the corrugated core in the x-direction. Fig. 2a shows schematically a unit-cell of corrugated core containing half period of the sinusoidal shape p and height of h_c. The unit-cell was originated at the lowest clamped end point and the width of the core was assumed to be unit,

$b = 1$. The position of a point on the middle surface can be considered as follows:

$$f(x) = \left(h_c - h_c \cos \frac{\pi x}{p}\right) \tag{1}$$

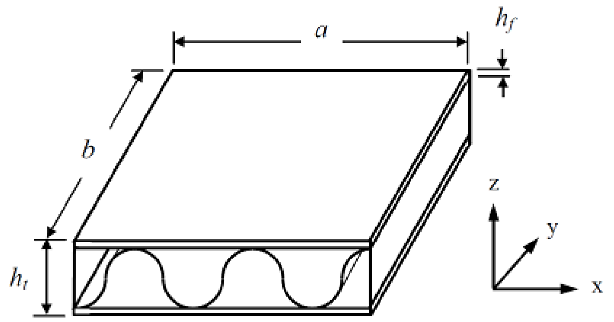

Fig. 1. Schematic representation of sandwich panel with corrugated core.

As represented in Fig. 2b, by applying a vertical force, V, in the z-direction to the upper end of core with the sinusoidal shape, positive vertical displacement, δ_V, and negative horizontal displacement in the other direction, δ_H , occurs. It is crucial to mention that a dummy moment, M_0, should be applied to the upper end of unit-cell to avoid rotation in the xz-plane.

The produced bending moment, normal and tangential forces at a distance x are as follows:

$$M(x) = xV - M_0 \tag{2a}$$

$$N(x) = -\sin \varphi V \tag{2b}$$

$$T(x) = \cos \varphi V \tag{2c}$$

where φ indicates angle of tangent in the point x. Applying the Castiglianos second theorem, the rotation of the upper end and its vertical and horizontal displacements can be derived as follows:

$$\delta_{M_0} = \int_0^p \left(\frac{M}{EI}\frac{\partial M}{\partial M_0} + \frac{N}{EA}\frac{\partial N}{\partial M_0} + \frac{T}{GA'}\frac{\partial T}{\partial M_0}\right)\frac{dx}{\cos\varphi} \tag{3a}$$

$$\delta_V = \int_0^p \left(\frac{M}{EI}\frac{\partial M}{\partial V} + \frac{N}{EA}\frac{\partial N}{\partial V} + \frac{T}{GA'}\frac{\partial T}{\partial V}\right)\frac{dx}{\cos\varphi} \tag{3b}$$

$$\delta_H = \int_0^p \left(\frac{M}{EI}\frac{\partial M}{\partial H} + \frac{N}{EA}\frac{\partial N}{\partial H} + \frac{T}{GA'}\frac{\partial T}{\partial H}\right)\frac{dx}{\cos\varphi} \tag{3c}$$

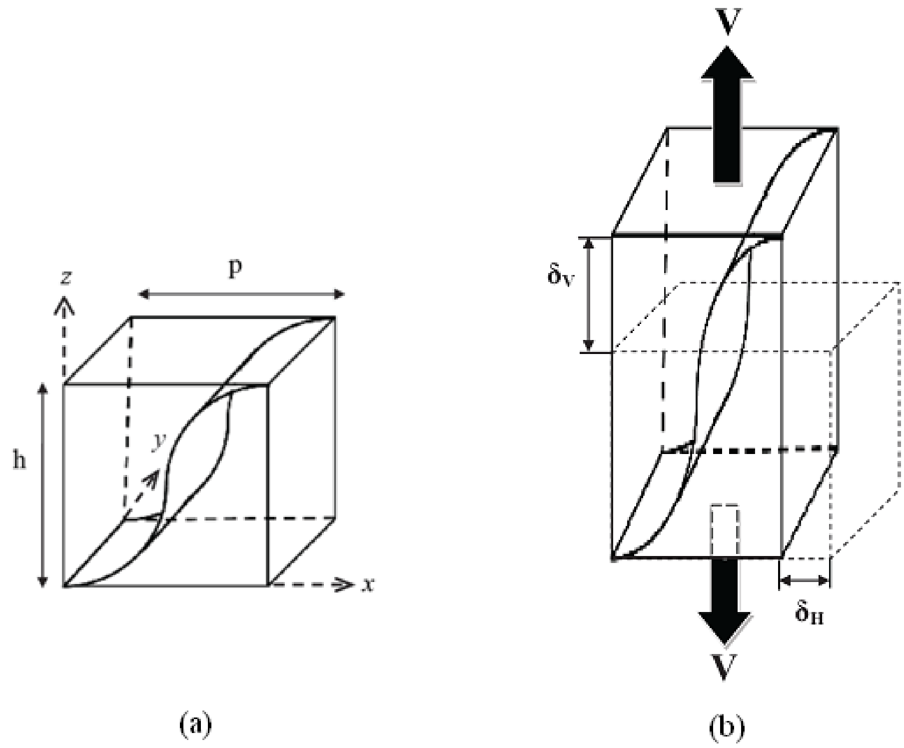

Fig. 2. A unit-cell of sinusoidal corrugated core: (a) before deformation (b) deformation along and orthogonal to the corrugation direction.

Solving Eqs. (3a) and (3b) by considering two conditions $V = 1$ and $\delta_{M_0} = 0$ and then substituting the results in Eq. (3c), one can obtain:

$$\delta_V = C_{22} - \frac{C_{23}^2}{C_{33}} \tag{4a}$$

$$\delta_H = C_{12} - \frac{C_{13}C_{23}}{C_{33}} \tag{4b}$$

The equivalent elastic modulus in the z-direction is then as follows:

$$E_z = \frac{\sigma_z}{\varepsilon_z} = \left(\frac{F_z}{A_{xy}}\right) \Big/ \left(\frac{\delta_V}{I_z}\right) = \frac{2h_c}{p\delta_V} \tag{5}$$

To obtain the Poisson's ratio, ν_{zx}, the ratio of lateral strain should be calculated along x-direction and the axial strain along z-direction. Thus:

$$\begin{aligned}\nu_{zx} &= -\frac{\varepsilon_x}{\varepsilon_z} \\ &= \left(\frac{\delta_H}{p}\right) \Big/ \left(\frac{\delta_V}{2h_c}\right) \\ &= \frac{2h_c}{p}\left[C_{12} - \frac{C_{13}C_{23}}{C_{33}}\right] \Big/ \left[C_{22} - \frac{C_{23}^2}{C_{33}}\right]\end{aligned} \tag{6}$$

where C_{ij} is a symmetric matrix introduced in Appendix.

3. Governing Equations

In the present section, the corrugated sandwich panel is considered as a laminated composite panel consisted of two facing plates and equivalent thick orthotropic layer instead of a corrugated core. In the absence of body forces, the three-dimensional equations of equilibrium for each layer of three-layer orthotropic plates are:

$$\begin{aligned}&\frac{\partial\sigma_x}{\partial_x} + \frac{\partial\tau_{xy}}{\partial y} + \frac{\partial\tau_{xz}}{\partial z} = 0, \\ &\frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\sigma_y}{\partial y} + \frac{\partial\tau_{yz}}{\partial z} = 0, \\ &\frac{\partial\tau_{xz}}{\partial x} + \frac{\partial\sigma_{yz}}{\partial y} + \frac{\partial\tau_z}{\partial z} = 0\end{aligned} \tag{7}$$

The constitutive equations for an orthotropic layer are as follows:

$$\{\sigma\} = [c]\{\varepsilon\} \tag{8}$$

where $\{\sigma\} = \{\sigma_x \ \sigma_y \ \sigma_z \ \tau_{zy} \ \tau_{zx} \ \tau_{xy}\}^T$, $\{\varepsilon\} = \{\varepsilon_x \ \varepsilon_y \ \varepsilon_z \ \gamma_{zy} \ \gamma_{zx} \ \gamma_{xy}\}^T$ and material properties matrix [c] is:

$$[c] = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \tag{9}$$

Note that, for corrugated core, the equivalent three-dimensional properties should be used in the above matrix; while for top and bottom faces only two independent isotropic engineering constants, i.e., E and ν are used.
The linear relations between the strain components and the displacements are as follows:

$$\begin{aligned}\varepsilon_x &= \frac{\partial u}{\partial x} & \gamma_{zy} &= \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \\ \varepsilon_y &= \frac{\partial u}{\partial y} & \gamma_{zx} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \\ \varepsilon_z &= \frac{\partial u}{\partial z} & \gamma_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\end{aligned} \tag{10}$$

The mechanical boundary conditions for outer surfaces of the sandwich panel are considered as follows:

$$\begin{aligned}&\sigma_z = \tau_{zx} = \tau_{zy} = 0 \quad \text{at} \quad z = 0 \\ &\sigma_z = q = q^* = \sin\left(\frac{\pi x}{a}\right) \cdot \sin\left(\frac{\pi y}{b}\right), \\ &\tau_{zx} = \tau_{zy} = 0 \quad \text{at} \quad z = h\end{aligned} \tag{11}$$

4. Exact Solution for Stress Field

For a corrugated sandwich panel with four simply supported edges, the following boundary conditions should be satisfied:

$$\begin{aligned}&u = w = 0, \quad \sigma_y = 0 \quad \text{at} \quad y = 0, b \\ &v = w = 0, \quad \sigma_x = 0 \quad \text{at} \quad x = 0, a\end{aligned} \tag{12}$$

The exact solution can be obtained from using the following solutions:

$$\begin{aligned}u(x,y,z) &= u^* \cos(q_n \cdot x) \cdot \sin(p_m \cdot y) \\ v(x,y,z) &= v^* \sin(q_n \cdot x) \cdot \cos(p_m \cdot y) \\ w(x,y,z) &= w^* \sin(q_n \cdot x) \cdot \sin(p_m \cdot y) \\ \sigma_x(x,y,z) &= \sigma_x^* \sin(q_n \cdot x) \cdot \sin(p_m \cdot y) \\ \sigma_y(x,y,z) &= \sigma_y^* \sin(q_n \cdot x) \cdot \sin(p_m \cdot y) \\ \sigma_z(x,y,z) &= \sigma_z^* \sin(q_n \cdot x) \cdot \sin(p_m \cdot y) \\ \tau_{zy}(x,y,z) &= \tau_{zy}^* \sin(q_n \cdot x) \cdot \cos(p_m \cdot y) \\ \tau_{zx}(x,y,z) &= \tau_{zx}^* \cos(q_n \cdot x) \cdot \sin(p_m \cdot y) \\ \tau_{xy}(x,y,z) &= \tau_{xy}^* \cos(q_n \cdot x) \cdot \cos(p_m \cdot y)\end{aligned} \tag{13}$$

where $p_m = \frac{m\pi}{b}$, $q_a = \frac{n\pi}{a}$ and $u^*, v^*, w^*, \sigma_x^*, \sigma_y^*, \sigma_z^*, \tau_{zy}, \tau_{zx}^*, \tau_{xy}^*$ are functions of z. By substituting relations Eq. (13) into Eqs. (7) and (8), the state-space equations can be obtained as follows:

$$\frac{d}{dz}\{\delta\} = G\{\delta\} \quad (14)$$

where $\{\delta\} = \{\sigma_z^* \ u^* \ v^* \ w^* \ \tau_{zx}^* \ \tau_{zy}^*\}^T$. G is the coefficients matrix presented in Appendix. The general solution to Eq. (14) explicitly expressed as:

$$\{\delta\} = e^{\int_0^z G\cdot dz}\{\delta_0\} = e^{G\cdot z}\{\delta_0\} \quad (15)$$

where $\{\delta_0\}$ is the state vector at the lowest surface. Imposing surface boundary conditions at the upper and lower surface, Eq. (11), the following equations can be obtained:

$$\begin{bmatrix} [A] \\ [S]^H \end{bmatrix}\{\delta_0\} = \{0\ 0\ 0\ -q\ 0\ 0\}^T \quad (16)$$

where matrix $[S]^H$ is and $[A]$ is given in Appendix. By solving Eq. (16), the $\{\delta_0\}$ was obtained. Then, application of Eq. (15) yields the state variables in other layers.

5. Results and Discussion

In an effort to illustrate the foregoing analysis, a simply supported sandwich panel with corrugated core made of aluminum was considered. It is worth noting that for the in-plane equivalent elastic constants all values were compared with Ref. [6] and all of them were exactly the same as the mentioned reference.

To verify the accuracy and reliability of the present approach, an aluminum sandwich panel with following geometrical parameters was considered: $h_f = 2t_c = 0.064''$, $p = 1.37'', hc = 0.75''$. The extensional and bending stiffness of corrugated sandwich panel were calculated and compared with both analytical and experimental results in Table 1. Experimental results are obtained due to 4-point bending test. Good agreement with the corresponding predictions from presented model can be observed. Since in-plane equivalent elastic constants of corrugated core have been presented analytically in the same previous researches, only equivalent out-of-plane constants, i.e. E_z, G_{xz}, G_{yz} and $_{xz}$ were plotted.

Table 1
Comparison of extensional and bending stiffness for sandwich panel.
$\left(h_f = 0.064'',\ h_c = 3/4'',\ t_c = 0.032'',\ E_c = 10.3\text{ksi},\ E_f = 10.5\text{ksi},\ \nu_f = \nu_c = \frac{1}{3},\ p = 1.37''\right)$

		$E_x(10^4)$	$E_y(10^4)$	$G_{xy}(10^4)$	ν_{xy}
Extensional stiffness	Present 3D analysis	157.0	196.9	73.8	0.27
	Classical theory†	186.9	138.7	58.3	0.25
		$D_x(10^4)$	$D_y(10^4)$	$D_{xy}(10^4)$	ν_{xy}
Bending stiffness	Present 3D analysis	25.1	26.9	18.2	0.31
	Classical theory†	25.0	22.0	16.7	0.29
	Experimental results†	-	22.1-22.4	18.2	-

† Libove and Hubka [1]

The Young's modulus and Poisson's ratio are $E = 71$ GPa and $= 0.33$, respectively. Based on industrial applications, for parametric studies, the initial values are considered as follows: $t_c = h_f = 0.001''$, $p = 1''$ and $h_c = 1/4''$.

Fig. 3 shows the out-of-plane equivalent elastic constants of corrugated sandwich panel versus non-dimensional ratio p/h_c for four values of h_c/t_c ratios. It is obvious that, in constant h_c/t_c ratio, Increasing the p/h_c will cause a significant decrease in the E_z and G_{yz} and notable decrease in the G_{xz}. On the other hand, under constant p/h_c ratio, the decline of core sheet thickness, tc to height of core hc (i.e. h_c/t_c) decreases the all equivalent out-of-plane modulus in the z-direction. On contrast, xz is not influenced by changing both p/h_c and h_c/t_c ratios.

In the figures to follow, z coordinate, displacement and stress components are non-dimensionalized as:

$$\eta = \frac{z}{h_t} - 0.5, \qquad \overline{U} = \frac{u^*}{h},$$
$$\overline{V} = \frac{v^*}{h}, \qquad \overline{W} = \frac{w^*}{h},$$
$$\overline{\sigma}_i = \frac{\sigma_i^*}{q_0}, \qquad \overline{\tau}_y = \frac{\tau_{ij}^*}{q_0}$$

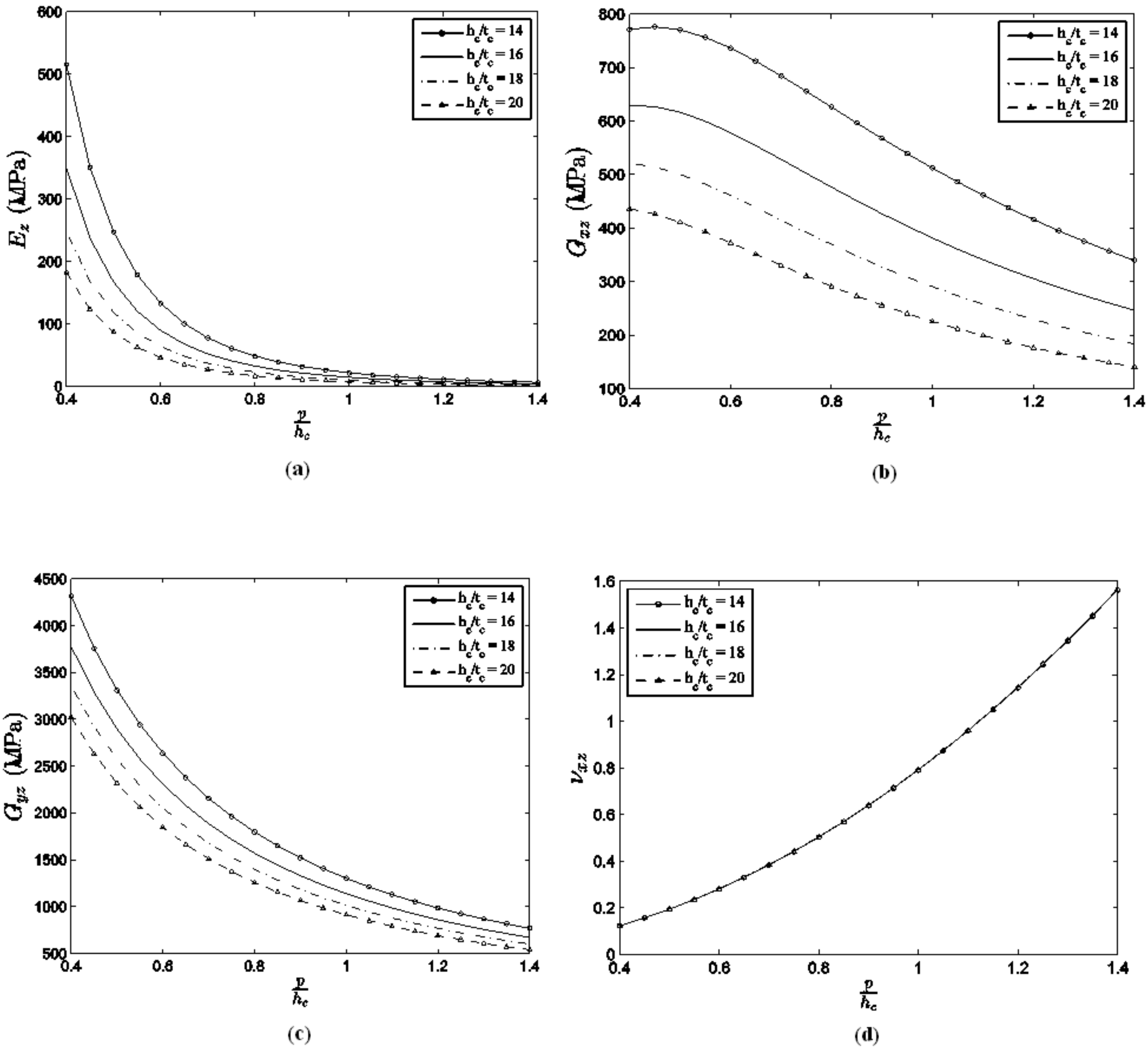

Fig. 3. Out-of-plane equivalent elastic constants of core against non-dimensional ratio, $p\Big/h_c$, for various $h_c\Big/t_c$ ratios. $\left(\dfrac{t_c}{t_f} = 1 \quad h_c = 1/4''\right)$

Fig. 4a-4f depicts through-thickness distributions of the dimensionless transverse displacements, $\overline{U}, \overline{V}, \overline{W}$ and normal and shear stresses $\overline{\sigma}_z, \overline{\tau}_{xz}, \overline{\tau}_{yz}$ with various h_c to t_c ratios. As would be anticipated, continuity was satisfied for all displacements and stresses; but sudden change in the slope of curves at the interface between the core and facing sheets can be observed. This is due to the fact that equivalent elastic constants of corrugated core and facing sheets are dissimilar which constitute discontinuity in the curve slopes.

From these figures, it can be observed that for larger $h_c\Big/t_c$ ratio, maximum in-plane displacements $\overline{U}$ and $\overline{V}$ will be increased. It was also indicated that the in-plane displacements across the thickness were very small and negligible in comparison to transverse displacement, $\overline{W}$. Furthermore, it was found that increasing of dimensionless $h_c\Big/t_c$ ratio causes a significant decrease in the transverse displacement.

The distribution of $\overline{\sigma}_z, \overline{\tau}_{zx}$ and $\overline{\tau}_{zy}$ through the thickness is depicted in Fig. 4d-4f. As shown in figures, change of $h_c\Big/t_c$ ratio has slight effect on $\overline{\sigma}_z$. It is obvious that the $\overline{\tau}_{zx}$ and $\overline{\tau}_{zy}$ first increases to a maximum value occur in the core and then decreases along the thickness. Increasing ratio of $h_c\Big/t_c$ will cause more nonlinearity in transverse stress distributions, $\overline{\tau}_{xz}$ and $\overline{\tau}_{yz}$. Moreover, inter-laminar stresses between corrugated core and facings are increased which can cause layer debonding.

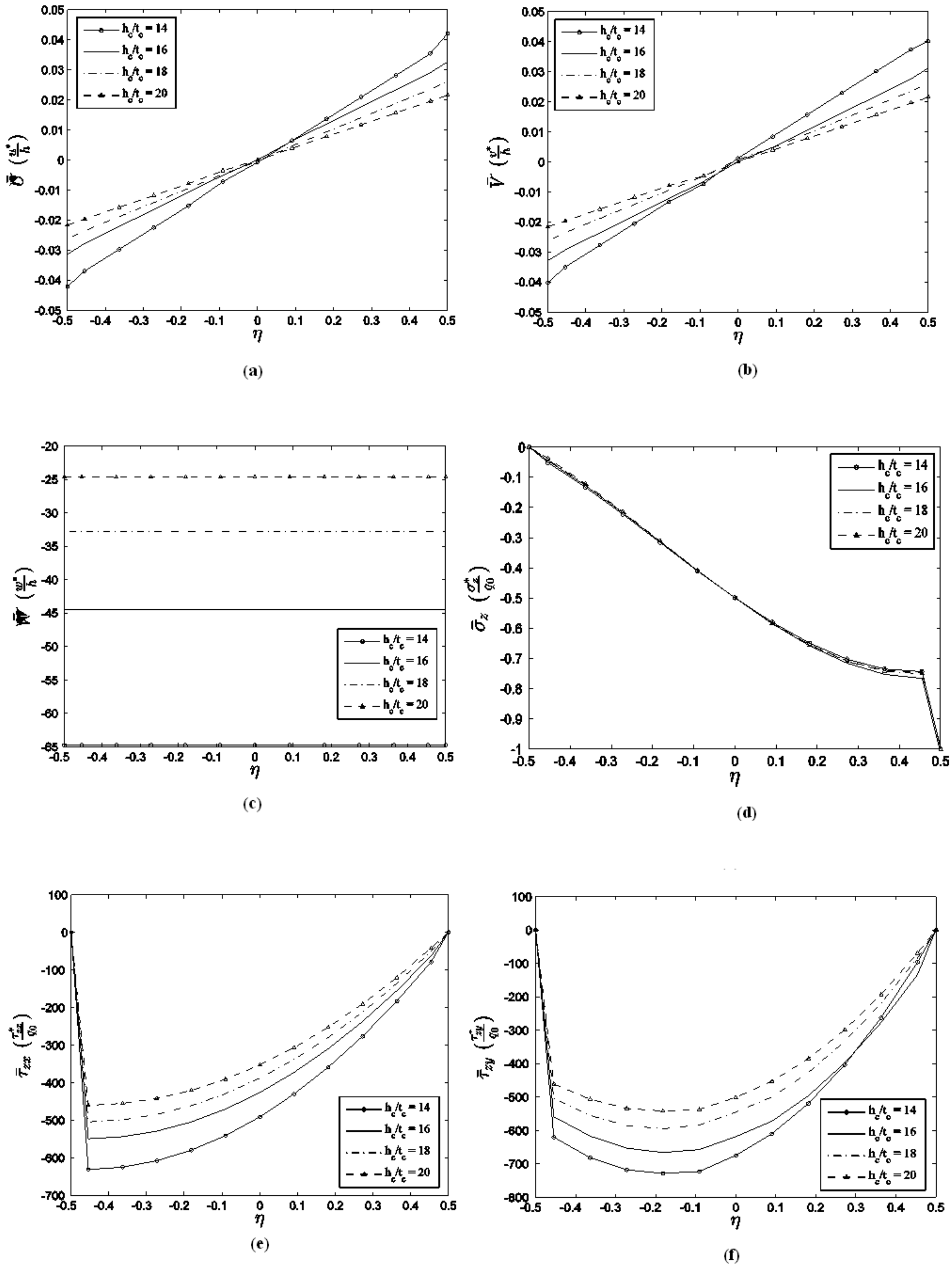

Fig. 4. Influence of non-dimensional ratio, $h_c\Big/t_c$, on the displacements and stresses of corrugated sandwich panels. ($h_c = 1/4''$)

Fig. 5 shows the effect of two parameters p/h_c and h_c/t_c, on the non-dimensional transverse displacement, $\overline{W}$. As expected, the curve shows clearly that the smaller the pitch-to-depth of core ratio is, the higher the transverse displacement will be. Also, transverse displacement tends to decrease with the increase of the h_c/t_c.

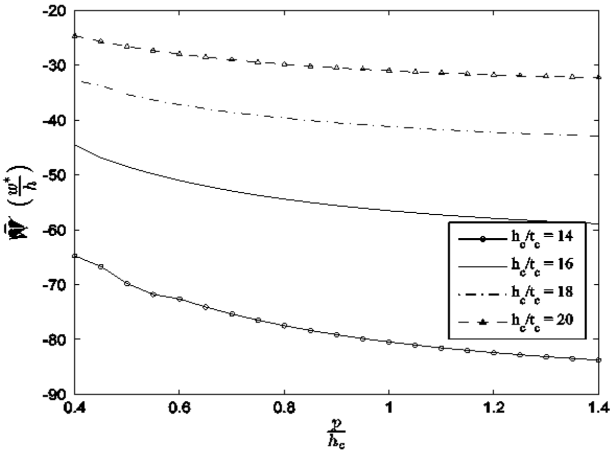

Fig. 5. Transverse displacement of sandwich panel against non-dimensional ratio p/h_c for various h_c/t_c ratios. $\left(x = \frac{a}{2} = 1, y = \frac{b}{2}, z = \frac{h}{2}, \frac{t_c}{t_f} = 1 \quad h_c = 1/4''\right)$

6. Conclusions

In this study, analytical formulations were proposed for sinusoidal corrugated sandwich panels. The energy method was used to obtain the main equivalent out-of-plane parameters. The governing equations of non-homogeneous, orthotropic laminated plates with equivalent properties for corrugated core were derived and put in a state-space matrix. Parametric studies were conducted for corrugated plates with different pitches, thicknesses and corrugation heights. It was found that out-of-plane properties of corrugated sandwich panel weaken with increasing pitch and height of corrugations, and strengthen when sheet of core becomes thicker. Also, the influences of these parameters on the stress and displacement fields were studied by using three-dimensional elasticity relations. It was found that out-of-plane properties can play an important role in characterizing the mechanical responses of corrugated sandwich panels which were omitted in the previous studies.

Appendix

$$[c] = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ & c_{22} & c_{23} \\ \text{sym.} & & c_{33} \end{bmatrix}$$

$$c_{11} = \frac{12}{t_c^2}\int_0^p t^2\sqrt{1+f'^2}dx + \int_0^p \frac{1}{\sqrt{1+f'^2}}dx + \frac{2(1+\nu)}{5/6}\int_0^p \frac{f'^2}{\sqrt{1+f'^2}}dx$$

$$c_{12} = \frac{12}{t_c^2}\int_0^p xf\sqrt{1+f'^2}dx + \left(\frac{2(1+\nu)}{5/6} - 1\right)\int_0^p \frac{f'}{\sqrt{1+f'^2}}dx$$

$$c_{13} = -\frac{12}{t_c^2}\int_0^p f\sqrt{1+f'^2}dx$$

$$c_{22} = \frac{12}{t_c^2}\int_0^p x^2\sqrt{1+f'^2}dx + \int_0^p \frac{f'^2}{\sqrt{1+f'^2}}dx + \frac{2(1+\nu)}{5/6}\int_0^p \frac{1}{\sqrt{1+f'^2}}dx$$

$$c_{23} = -\frac{12}{t_c^2}\int_0^p x\sqrt{1+f'^2}dx$$

$$c_{33} = -\frac{12}{t_c^2}\int_c^p \sqrt{1+f'^2}dx$$

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 & q & p \\ 0 & 0 & 0 & -q & 1/c_{55} & 0 \\ 0 & 0 & 0 & -p & 0 & 1/c_{44} \\ 1/c_{33} & qc_{13}/c_{33} & pc_{23}/c_{33} & 0 & 0 & 0 \\ -qc_{13}/c_{33} & G_{52} & G_{53} & 0 & 0 & 0 \\ -pc_{23}/c_{33} & G_{62} & G_{63} & 0 & 0 & 0 \end{bmatrix}$$

where

$$G_{52} = q^2\left(c_{11} - \frac{c_{13}^2}{c_{33}}\right) + p^2 c_{66}$$

$$G_{53} = G_{62} = pq\left(c_{12} - \frac{c_{13}c_{23}}{c_{33}} + c_{66}\right)$$

$$G_{62} = p^2\left(c_{22} - \frac{c_{23}^2}{c_{33}}\right) + q^2 c_{66}$$

$$[A] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

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