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An Analytical Model for Long Tube Hydroforming in a Square Cross-Section Die Considering Anisotropic Effects of the Material

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Abstract

In this paper, a mathematical model was developed to analyze the hydroforming process of a long anisotropic circular tube into a square cross-section die. By using the thickness variation in two extreme cases of friction between the tube and die wall, namely no friction and sticking friction cases, thickness variation in the case of sticking friction was captured in the model. Then by using equilibrium equation for contact length segment, thickness distribution was determined and corresponding forming pressure is predicted. It was shown that in a plane strain state, anisotropic value has no influence on thickness variation of the deformed tube and the forming pressure will increase when the anisotropic value increases. The analytical results of forming pressures and thickness distributions were compared with the results available in the literature to verify the validity of this simple analytical proposed model.

Nomenclature

dL	contact length increment	P	forming pressure
R	anisotropic value	r_o	initial tube radius
r	radius of arc segment	L	length of linear segment, contact length
t_o	initial tube thickness	$t_s(x)$	deformed tube thickness in case of sticking at distance x of centerline
t_u	deformed tube thickness in case of frictionless	$t(x)$	deformed tube thickness in guided zoneat distance x of centerline
K	strength coefficient	n	strain hardening exponent
α	a parameter where $0 \leq \alpha \leq 1$	$\bar{\alpha}$	effective stress
$\bar{\varepsilon}$	effective strain	μ	friction coefficient
$\sigma_\theta, \sigma_z, \sigma_t$	hoop, longitudinal and thickness stress components, respectively	$\varepsilon_\theta, \varepsilon_z, \varepsilon_t$	hoop, longitudinal and thickness strain components, respectively

1. Introduction

Tube hydroforming is a process in which a relatively thin-walled circular tube is expanded by an internal pressure and is forced to conform to a shaped die that surrounds it. The tube first elastically and then plastically deforms in the die. The deformation proceeds with an elevation of contact length between the tube and the die as the pressure is increased. Conventionally, the pressure and the distribution of thickness of this shell along its periphery are unknowns that depend on the geometry, friction coefficient between the tube and the die and material properties. It is desirable, therefore, to develop some simple but efficient meth-

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ods to facilitate the pressure and thickness variation prediction in tube hydroforming process [1]

Some of the analytical models and various studies using analytical models and the finite element method have been conducted on plane strain tube hydroforming. Chen et al. [1] used finite element simulation to study the effects of forming pressure and coefficient of friction on corner filling in tube hydroforming. Kridli et al. [2] used finite element simulation to study the effects of material strain hardening and friction coefficient on the thickness distribution. The two-dimensional hydroforming of circular tubes into rectangular sections was studied by Hwang and Altan [3] using finite element method. Experimental measurements of the coefficient of friction during hydroforming were carried out by Vollertsen and Plancak [4]. Their approach depended on correlating thickness variation along the tube axis with friction coefficient when upsetting an internally pressurized tube. Chen et al. [1] modelled hydroforming of a round tube inside a square die cavity using finite element code, namely ABAQUS/Standard. Kridli et al. [5] reported the results of two-dimensional plane strain finite element models of the tube hydroforming using a square die, which were conducted using the commercial finite element code ABAQUS/Standard.

The effects of the strain-hardening exponent, initial tube wall thickness, and the die corner radii on corner filling and thickness distribution of the hydroformed tube were discussed. Characteristics of corner filling using a square die have been investigated by Liu et al. [6] via mechanical analyses and numerical simulations. Hwang and Chen [7, 8] developed a mathematical model to analyze tube expansion in a rectangular die with sticking friction condition and a square cross-section die with sliding friction condition.

In 2007, Orban and Hu [9] developed an analytical model to determine the variation in the stresses and strains along the tube wall as the forming pressure increases to expand the circular tube into a square cross-section die. The friction dependency of the corner filling with frictionless, sliding, and sticking friction conditions was studied. Miller et al. [10, 11] developed a 2D analytical model capable of capturing the effects of tension, pressure, the effects of tension, pressure and loading history on the quality of the tubes. The results from this model were in good agreement with experimental results, implying that the 2D analytical model can be an efficient tool for parametric study of the forming process at the design stage. Corona [12] extended the model mentioned above to be applicable for arbitrary tube cross-sections. Based on the work of Miller et al. [10] and Corona [12], Guan and Pourboghrat [13] proposed a Fourier-series-based FEA of tube hydroforming for axisymmetric model and Guan and Pourboghrat discussed generalized plane strain model [14]. Smith et al. [15] and Smith and Sun [16] introduced closed-form formula for planar tube hydroforming analysis. Yang and Ngaile [17] developed an analytical model for planar tube hydroforming based on deformation theory. The developed analytical model was used to predict hydroformed shape, corner fill, wall thinning, and forming pressure. Since the model was based on a mechanistic approach with bending effects included, local strain and stress distribution across the wall thickness could be determined.

In this paper, an analytical model considering interfacial friction between die and tube was developed to predict the forming pressure, needed to expand a circular long anisotropic tube into a square cross-section die, and the thickness distribution of the product. There was no axial feed and the forming process was under a plane strain state along the axial direction. The proposed solution was based on this fact that the thickness variation in the case of sliding friction remains between thickness variation of two extreme cases of friction, namely no friction and sticking friction. The analytical results of forming pressure and thickness distributions of the formed tube were compared with the available published results. In spite of the simplicity of the proposed solution, the relative difference between this solution and the solutions based on complicated approaches was comparatively small. This is a great advantage for engineering applications.

2. Analytical Model

2.1. Thickness Variation Assumption

The schematic diagram of a circular long tube that initially just fit inside the square cross-section die is shown in Fig. 1. Taking into account the symmetry of the problem, only one-eighth of the initial tube was considered. In this figure, t_o and r_o are the initial thickness and outer radius of the tube, respectively. As it is shown in the figure, at any given stroke of the deformation, there are two segments in the deformed tube, the guided zone (contact length between deformed tube and the die) and the free expansion segment. When internal pressure is applied, the tube deforms such that the arc length decreases while the contact length between the tube and the rigid die increases. As contact length progresses, the free expansion segment of the tube will stretch and will contact the die wall. It was assumed that the profile of the free expansion segment is a circular arc and the centre of the arc is at the origin O. At any given length of linear segment, contact length= L, the arc radius can be determined as $r = r_o - L$.

If the die wall is frictionless, the thickness will be uniformly distributed all over the deformed tube and a tube with a constant thickness would be formed. Friction at the die wall restricts the flow of material in contact with the die and causes a variation in the thickness of the formed tube along the cross-section with the smallest thickness at the corners. Thus the frictional behavior at the material/die interface is crucial in tube hydroforming. To study the friction effects, a simple model and corresponding procedure were proposed. In this work, two extreme cases, with no friction at the die wall or with sticking friction along the entire contact length were considered. During the formulation, the following assumptions were employed:

1. the die is rigid.
2. there is no axial feed and it is assumed that plastic deformation of the tube is under a plane strain state along the axial direction.
3. the initial cross-section of the tube is perfectly circular and the initial thickness is uniform.
4. the friction coefficient is uniform all over the die surface.
5. during the deformation, the strain is assumed to be constant across the thickness.
6. the effect of lateral pressure on the flow stress is neglected.
7. the material is transversely anisotropic.

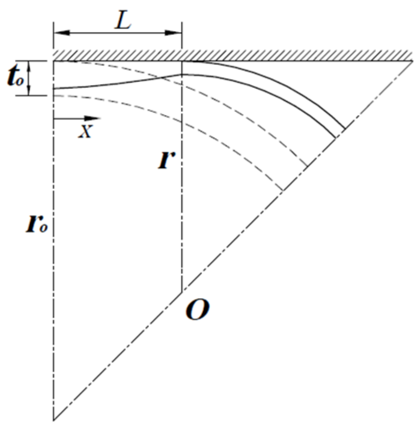

Fig. 1. One–eighth of the initial (dashed line) and deformed tube (continuous line).

If contact between the tube and the die is frictionless, the guided zone will have the same strain as that in the arc segment and at any instant the thickness at any point around the tube will be uniform and we have [18]:

$$t_u(L) = \frac{t_o}{\dfrac{4}{\pi} - \dfrac{L - r_o}{r_o}\left(\dfrac{4}{\pi} - 1\right)} \tag{1}$$

For a very high friction coefficient, the guided zone will stick to the die and will not experience further straining. If the tube sticks to the die wall as soon as it touches it, then the thickness at the first point of contact will be t_o. In the sticking friction mode, the materials after contact with the die do not move or slide. As the tube becomes progressively attached to the die wall, the thickness will decrease and at point with distance from axis of symmetry, as shown in Fig. 1, it has the value [18]

$$t_s(x) = t_o\left(\frac{r_o - x}{r_o}\right)^{\frac{4}{\pi}-1} \tag{2}$$

The deformed tube thickness in guided zone will be non-uniformly varying from the initial thickness at the axis of symmetry of the die to a minimum at the arc segment. For a given radius r, the arc thickness will be less than that for the frictionless case as shown in Fig. 2.

In order to take the friction into account, the proposed solution was based on this fact that the thickness variation in the case of sliding friction remains between thickness variation of two extreme cases of friction, i.e. no friction and sticking friction. It was assumed that the thickness distribution in the linear segment and the contact length is approximated as follows:

$$t(x) = t_u(L) + \alpha[t_s(x) - t_u(L)] \tag{3}$$

where $t(x)$ is tube thickness at distance of the axis of symmetry, $t_u(L)$ and $t_s(x)$ can be given by Eqs. (1) and α (2), respectively and is a parameter where $0 \leq \alpha \leq 1$. The distribution of thickness in the linear segment, contact length, can be determined by finding an appropriate value for using force equilibrium of the linear segment.

Thickness variation along guided zone of deformed tube for frictionless, sticking and friction cases is shown in Fig. 2. When contact between the tube and the die wall is frictionless, α is equal to zero and when full sticking prevails at the interface, α is equal to 1 and for intermediate friction case $0 < \alpha < 1$.

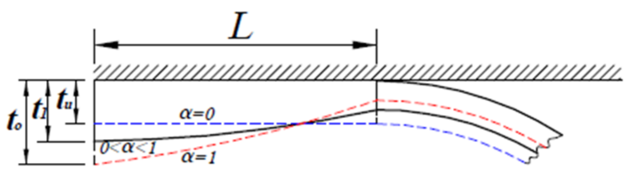

Fig. 2. Thickness variation along contact length for frictionless ($\alpha = 0$), sticking ($\alpha = 1$) and intermediate friction cases ($0 < \alpha < 1$).

2.2. Stress and Strain Analysis

According to the principle of volume constancy and the assumption of plane strain state, the equivalent strain and the equivalent stress of an element can be, respectively obtained from Hills yield criterion as

$$\bar{\varepsilon} = \sqrt{\frac{2(R+2)(R+1)}{3(2R+1)}}\varepsilon, \tag{4}$$

$$\bar{\sigma} = \sqrt{\frac{3(R^2+R+1)}{(R+1)^2(R+2)}}\sigma_\theta \tag{5}$$

where is the anisotropic value and is σ_θ the hoop stress. Assuming that the flow stress of the tube material follows the power law of its equivalent strain, the equivalent stress is denoted as

$$\bar{\sigma} = K\bar{\varepsilon}^n \tag{6}$$

where K and n are strength coefficient and strain-hardening exponent, respectively. At each contact length L, forming pressure P was approximated through consideration of thin-walled cylindrical tube in which the walls offered little resistance to bending and a uniform stress distribution in the wall could be assumed. Then

$$P = \frac{\sigma_{\theta 2} t_t}{r} \tag{7}$$

where r is radius of arc, t_2 and $\sigma_{\theta 2}$ are the tube thickness and hoop stress component at the arc segment, respectively.

2.3. Equilibrium of Linear Segment

For a given tube contact length L, the shear stress at the die wall was μP. Taking summation of forces acting on the guided zone in the direction, as shown in Fig. 3, we obtain

$$\sum F(x) = \sigma_{\theta 2} t_2 - \mu PL \tag{8}$$

where μPL is the friction force at the material/die interface per unit width of the tube. Appropriate value of parameter α was given by satisfying the above equilibrium equation for linear segment. For a given value of α, tube thickness at $x = 0$, t_1, and at $x = L$, t_2, can be given by Eq. (3). Then, thickness strain at $x = 0$, ε_{t1}, and at $x = L$, ε_{t2}, can be determined as

$$\varepsilon_{t1} = \ln\frac{t_o}{t_1}, \qquad \varepsilon_{t2} = \ln\frac{t_o}{t_2} \tag{9}$$

Corresponding hoop stress components at $x = 0$, $\sigma_{\theta 1}$, and at $x = L$, $\sigma_{\theta 2}$, can be given by Eqs. (5) and (6) as

$$\sigma_{\theta 1} = K\sqrt{\frac{3(R^2+R+1)}{(R+1)^2(R+2)}}\left(\sqrt{\frac{2(R+2)(R+1)}{3(2R+1)}}\varepsilon_{t1}\right)^n \tag{10}$$

$$\sigma_{\theta 2} = K\sqrt{\frac{3(R^2+R+1)}{(R+1)^2(R+2)}}\left(\sqrt{\frac{2(R+2)(R+1)}{3(2R+1)}}\varepsilon_{t2}\right)^n \tag{11}$$

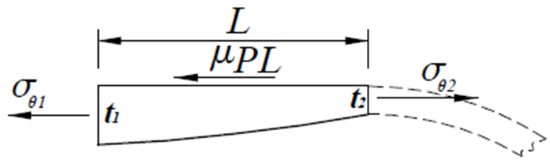

Fig. 3. Free body diagram of the stresses acting on tube contact length L.

Substituting Eqs. (10) and (11) into equilibrium Eq. (8), for a given contact length and a value of between 0 and 1, one of the two following conditions will occur:

$$\sigma_{\theta 2} - \sigma_{\theta 1} t_1 - \mu PL = 0 \tag{12a}$$

$$\sigma_{\theta 2} - \sigma_{\theta 1} t_1 - \mu PL > 0 \tag{12b}$$

If $\sigma_{\theta 2} - \sigma_{\theta 1} t_1 - \mu PL = 0$ then, the material in the contact length is in equilibrium and the thickness distribution and forming pressure are determined by Eqs. (3) and (7), respectively. If $\sigma_{\theta 2} - \sigma_{\theta 1} t_1 - \mu PL > 0$ then a new material from the arc segment comes into contact with the die and the thickness of the arc segment decreases. At this condition, the value of should be increased by $d\alpha$ and the values of t_1, t_2, ε_{t1}, ε_{t2}, and $\sigma_{\theta 1}$ and $\sigma_{\theta 2}$, are updated using Eqs. (3) and (9) -(11), respectively. The value of increases until $\sigma_{\theta 2} - \sigma_{\theta 1} t_1 - \mu PL = 0$ and the current α is recorded. Then, for this value of α, thickness distribution and forming pressure are determined by Eqs. (3) and (7), respectively.

If for all values of α, the summation in Eq. (8) is negative, then full sticking (very high friction) is occurred. Material in the sticking zone undergoes no deformation and thickness distribution and forming pressure are determined by Eqs. (3) and (7), respectively for $\alpha = 1$.

It is obvious that when contact between the tube and the die wall is frictionless, α is equal to zero and with increase in the friction coefficient, parameter α will increase and when full sticking prevails at the interface, α is equal to 1.

2.4. Effect of the Anisotropy Value on Thickness Variation

To investigate the effect of the anisotropy value on thickness variation of the deformed tube, Eq. (7) is substituted into equilibrium Eq. (12a), then

$$\sigma_{\theta 2} t_2 - \sigma_{\theta 1} t_1 - \mu\frac{\sigma_{\theta 2} t_2}{r} L = 0 \tag{13}$$

Simplifying of the above equation yields

$$\sigma_{\theta 2} t_2 (r - \mu L) - \sigma_{\theta 1} r t_1 = 0 \tag{14}$$

Substituting Eqs. (10) and (11) into Eq. (14) we have

$$\frac{K\sqrt{\dfrac{3(R^2+R+1)}{(R+1)^2(R+2)}}\left(\sqrt{\dfrac{2(R+2)(R+1)}{3(2R+1)}}\varepsilon_{t1}\right)^n}{K\sqrt{\dfrac{3(R^2+R+1)}{(R+1)^2(R+2)}}\left(\sqrt{\dfrac{2(R+2)(R+1)}{3(2R+1)}}\varepsilon_{t2}\right)^n} t_2(r-\mu L) - rt_1 = 0 \tag{15}$$

Finally:

$$\frac{\varepsilon_{t2}^n}{\varepsilon_{t_1}^n} t_2(r-\mu L) - rt_1 = 0 \tag{16}$$

As shown in above equation, equilibrium equation is independent of R and therefore anisotropy value has no influence on the thickness variation of the formed tube in plane strain state.

3. Validation of the Analytical Model

The analytical model discussed in the previous section was aimed at predicting the forming pressure and thickness distribution along linear segment. A MATLAB program was implemented for the previously derived equations. Initial tube geometry, friction coefficient, and tube material properties were inputs of the computer program. The computer program calculates thickness variation and the corresponding required forming pressure for a given contact length, L. In this section, forming pressure and thickness distribution of the deformed tube obtained from the analytical model was compared with the results of [8] to verify the validity of this simple proposed model.

The forming conditions used in the analytical model of tube hydroforming in a square cross-section die were $r_0o = 30$mm, $t_0 = 2$mm. The die width was the same as the outside diameter of the tube. The flow stress of the tube material, AISI 1008 used in the analytical model was $\bar{\sigma} = 657.2\bar{\varepsilon}^{0.24}$MPa [9], tensile strength 340MPa, yield strength 285MPa, elongation 20% and anisotropic value $R = 1$. In order to obtain the friction coefficient between the tube and die, the so-called ring compression test was conducted by Ref. [8]. Four values of the friction coefficients used in the study were $\mu = 0.01$, 0.05, 0.1 and 0.5 [8]. Comparison between predicted values of forming pressure and those of Ref. [8] for various friction coefficients is shown in Fig. 4. As the forming pressure increased beyond yielding, the contact length between the die and the tube increased and the arc radius decreased. The results showed that the values of forming pressure had a very good agreement with the results of Ref. [8] and they are coincident with each other. From this figure, it is known that the effect of friction coefficient upon the forming pressure is not significant.

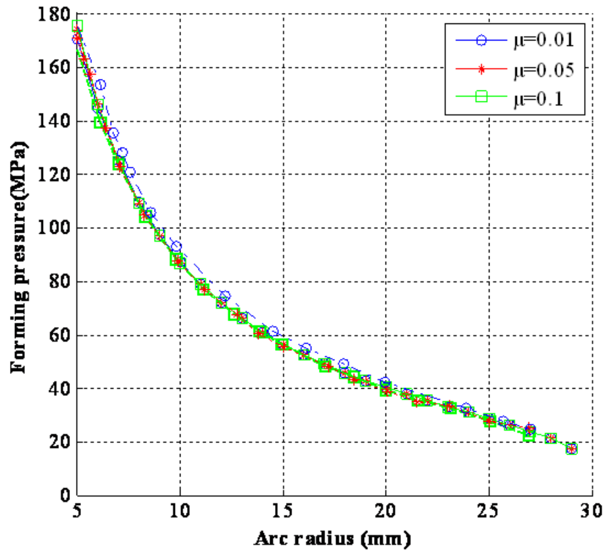

Fig. 4. Variation of forming pressure with arc radius for various frictions and $r_0 = 20$mm, $t_0 = 2$mm, $R = 1$. (Comparison between predicted values, continues line, and those of Ref. [8], dashed line)

As the deformation progressed the tube thickness decreased. The maximum thinning occured at the arc segment. Changes in arc thickness with contact length for various friction coefficients are shown in Fig. 5. As seen in this figure, as the friction increases it causes greater corner thinning.

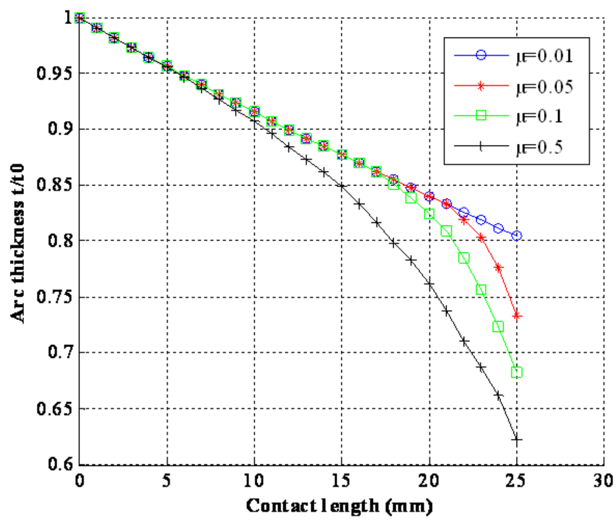

Fig. 5. Change in arc thickness for various friction coefficients for $r_0 = 30$mm, $t_0 = 2$mm, $R = 1$.

Effect of the friction coefficient upon the thickness variation along contact length is shown in Fig. 6. In this figure, predicted values and those of Ref. [8] are compared. For the frictionless case the wall thickness was uniform with a value of 0.82mm. A larger friction coefficient will result in a smaller thickness or a larger strain at the arc segment, thus, it is more likely to lead to failure of the tube for a larger friction coefficient at the die-tube interface. As the friction increases, the variation of the thickness increases. For $\mu = 0.05$, thickness varied from 0.74 to 0.89mm. Therefore, Thickness variation was 18%. While, for $\mu = 0.1$, thickness varied by 25%, from 0.69 to 0.92.

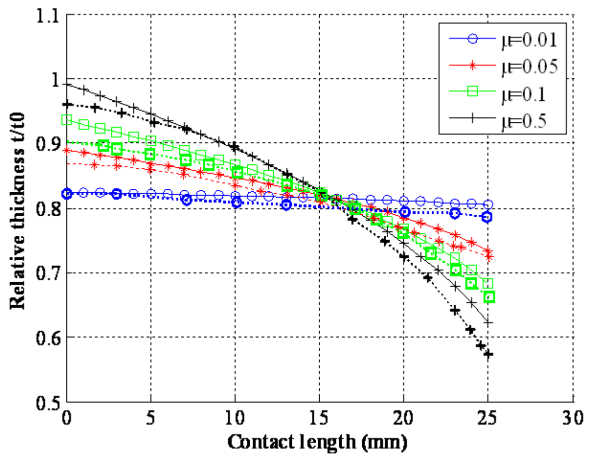

Fig. 6. Comparison of predicted thickness variations (continues lines) and values of Ref. [8] (dashed lines), for $r_0 = 30$mm, $t_0 = 2$mm, $R = 1$.

Effects of the strain-hardening exponent, $n = 0.14$ and $n = 0.24$, on the forming pressure are shown in Fig. 7. The friction coefficient was maintained at 0.1. As seen in this figure, as the strain-hardening exponent increases, it causes greater corner thinning and lower forming pressure.

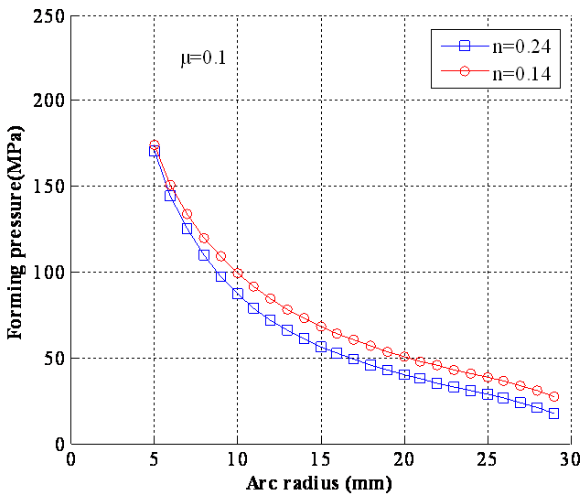

Fig. 7. Effect of the strain-hardening exponent on the forming pressure for $r_0 = 30$mm, $t_0 = 2$mm, $R = 1$.

In Fig. 8, the effect of the anisotropic coefficient on the forming pressure is shown. Anisotropic values are defined as the ratio of the strain in the width direction to that in the thickness direction. Form this figure it is known that a larger anisotropic value can raise the forming pressure, in other words, it becomes more difficult to deform the tube to a desired corner radius.

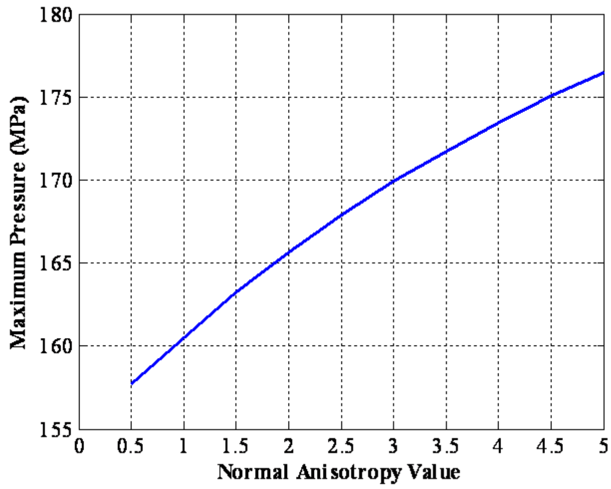

Fig. 8. Effect of the anisotropic value on the forming pressure for $r_0 = 30$mm, $t_0 = 2$mm, $\mu = 0.05$.

4. Conclusions

An analytical model was developed to study the effect of the various process parameters on the thickness distribution and forming pressure for circular long anisotropic tube hydroforming in a square cross-section die. By using the thickness variation in two extreme cases of friction between tube and die wall, no friction and sticking friction, thickness variation in the case of friction was determined in the model. It was shown that anisotropic value has no influence on thickness variation of the formed tube in plane strain state and the forming pressure will increase when anisotropic value increases.

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