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The Effect of Grading Index on Two-dimensional Stress and Strain Distribution of FG Rotating Cylinder Resting on a Friction Bed Under Thermomechanical Loading

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Abstract

This paper presents two-dimensional stress and strain behavior of a FG rotating cylindrical shell subjected to internal-external pressure, surface shear stresses due to friction, an external torque, and constant temperature field. A power law distribution was considered for thermomechanical material properties. First order shear deformation theory (FSDT) was used to define the displacement and deformation field. Energy method and Euler equation were employed to derive constitutive differential equations of the rotating shell. Systems of Six differential equations were achieved. Eigenvalue and eigenvector methods were used to solve these equations. It was found that the material grading index has a significant effect on stresses and strains of a rotating functionally graded material cylindrical shell in radial and longitudinal directions.

Nomenclature

r	Radius of an arbitrary layer of cylinder	R_i	Inner radius
z	Coordinate of arbitrary layer of cylinder respect to middle surface	V_θ	Circumferential component of deformation
R	Radius of mid-surface of cylinder	T_i	Inner temperature
U_r	Radial component of deformation	T_0	Outer temperature
W_x	Axial component of deformation	ε_{rr}	Radial strain
R_0	outer radius	ε_{xx}	Axial strain
u	Displacement component of radial deformation	w	Displacement component of axial deformation
$\varepsilon_{\theta\theta}$	Circumferential strain	γ_{rx}	Shear strain in rx plane
v	Displacement component of circumferential deformation	ϕ_z	Rotational component of radial deformation
$\gamma_{r\theta}$	Shear strain in $r\theta$ plane	$\gamma_{x\theta}$	Shear strain in $x\theta$ plane
ϕ_x	Rotational component of axial deformation	ϕ_θ	Rotational component of circumferential deformation
σ_{rr}	Radial stress	σ_{xx}	Axial stress
E	Modulus of elasticity	$\sigma_{\theta\theta}$	Circumferential stress

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U	Total energy	τ_{rx}	Shear stress in rx plane
P_i	Inner pressure	$\tau_{r\theta}$	Shear stress in $r\theta$ plane
P_0	Outer pressure	$\tau_{x\theta}$	Shear stress in $x\theta$ plane
F	General potential function	σ_{eff}	Effective stress

1. Introduction

Functionally graded materials (FGMs) are advanced composites in which element gradation changes continuously from metal to ceramic phase or vice versa depending on the requirements. Continuous changes in the FGM composition results in improving the mechanical and thermal properties. Bewar and Duwez [1] first explained and introduced the FGM concept theoretically. However, FGM was successfully developed by Japanese scientists in 1984 for an aerospace application to attain a thermal gradient of 1000°K along 10mm cross-section [2]. Since then FGM has drawn more attention in most of the engineering utilities such as carriage systems, medical application, nuclear components, space vehicle components, energy storage systems, aero engine components, thermal protection packages for high temperature environments and etc. [3]. Shells are structural elements widely used in applications such as mechanical, civil, aeronautical, and marine engineering. Shell structures are used as roofs, liquid storage vessels, nuclear plant accessories, piping structures, and pressure vessels [4].

Hollow composite cylindrical shells have many engineering uses because composite materials have tailoring properties, less weight, low maintenance, and high performance with increased service life. Functionally graded material is a type of composite appropriate for bulk and shell material application. With increasing usage of cylindrical shells and FGM in various applications, it is necessary to investigate the performance of FGM cylindrical shells at different working load conditions. Cylindrical shells mainly fail due to axial compressive stresses, buckling loads and large deformations due to internal pressure.

Various problems of FGM have attracted considerable attention in recent years. That is an important topic in engineering because of many rigorous applications. The study on the stresses and strains in rotating hollow cylinders has never stopped because of the importance of these basic elements in several mechanical, building, power and computer engineering applications. Loghman and Wahab [5] studied the thermo-elasto-plastic and residual stresses in thick-walled cylindrical pressure vessels of strain hardening material. Horgan and Chan [6] solved the classic problem of stress distribution in an inhomogeneous isotropic rotating solid disc and pressurized hollow cylinder.

Moradi et al. [7] investigated reverse yielding and the Baushinger effect on residual stresses in thick-walled cylinders. Tutuncu and Ozturk [8] calculated the stress distribution in an axisymmetric structure. They derived closed-form solutions for the stresses and deformations of functionally graded cylindrical and spherical shells under internal pressure. Ghorbanpour et al. [9] presented the Bauschinger and hardening effect on residual stresses in thick-walled cylinders of SUS 304. A computational study on functionally graded rotating solid shafts was carried out by argeso and Eraslan [10]. Displacements and stresses of rotating FGM thick hollow cylindrical shell under internal pressure and thermal load was studied by Zamani nejad and Rahimi [11].

Using plane theory of elasticity and procedure of complementary functions, Tutuncu and Temel [12] determined axisymmetric deformations and stresses in functionally graded hollow cylinders, disks, and spheres under uniform internal pressure. Eipakchi [13] derived stresses and displacements of a thick conical shell with variable thickness subjected to distributed nonuniform internal pressure analytically using third-order shear deformation theory (TSDT). Azturk and Galgec [14] studied elastic-plastic stress in a long functionally graded solid cylinder with fixed ends subjected to uniform heat generation. Khorshidvand and Javadi [15] investigated deformation and stresses in FG rotating hollow disk and cylinder Subjected to Thermal and Mechanical Load. Ghannad et al. [16] investigated elastic behavior of pressurized thick cylindrical shells with variable thickness made of functionally graded materials using FSDT. Zamani nejad et al. [17] studied the Effect of exponentially-varying properties on Displacements and Stresses in Pressurized Functionally Graded Thick Spherical Shells using Iterative Technique. Fatehi and Zamani nejad [18] considered the effects of material gradients on the onset of yield in fgm rotating thick cylindrical shells. Zamani nejad and Gharibi [19] considered the effect of material grading index on stresses of thick FGM spherical pressure vessels with exponentially-varying properties.

Jabbari et al. [20] studied the effect of material gradient on stresses of FGM rotating thick-walled cylindrical shell with longitudinal variation of properties under non-uniform internal and external pressure. Arefi et al. [21] investigated the effect of axially variable thermomechanical loads on the 2D thermo-elastic response of FG cylindrical shell. Singh et al. [22] investigated stress and deformation of rotating cylindrical pressure vessel of functionally graded material modeled by Mori-Tanaka scheme. Habibi et al. [23] evaluated the stress intensity factor (SIF) in FGM thick-walled cylindrical vessel. Jabbari et al. [24] considered the analysis of stress in rotating thick truncated conical shells with variable thickness under thermomechanical loads.

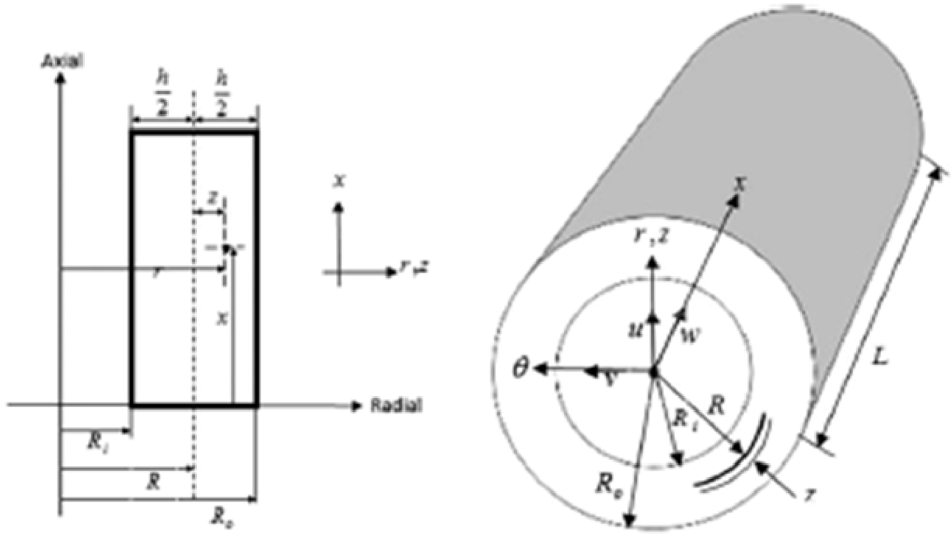

Fig. 1. A rotating cylinder made of FGM material and the selected coordinate system.

From the abovementioned literature, one can realize that a rotating cylinder under an applied external torque on a friction bed has not yet been considered.

The main objective of this paper is stress and strain analysis of a finite length rotating FG cylinder subjected to a distributed shear stress due to outer surface friction, an external driving torque, internal-external pressure and a uniform temperature distribution using FSDT.

2. Temperature Distribution

For the cylindrical shell in this study a steady state symmetrical conduction heat transfer without heat generation was considered. The reduced heat conduction equation in this case is written [25] as follows:

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(kr\frac{\mathrm{d}T}{\mathrm{d}r}\right)=0 \tag{1}$$

Using the power law distribution for the material thermal conductivity coefficient, ($k_T = r^{\beta}k$) the above equation can be written as:

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(r^{\beta+1}k\frac{\mathrm{d}t}{\mathrm{d}r}\right)=0 \quad\Rightarrow\quad r^{\beta}k\frac{\mathrm{d}T}{\mathrm{d}r}+r^{\beta+1}k\frac{\mathrm{d}^2T}{\mathrm{d}r^2}=0, \tag{2}$$

$$T(r)=-A_1r^{-\beta}+A_2 \tag{3}$$

$$A_1=\frac{T_i-T_0}{-r_i^{-\beta}+r_0^{-\beta}} \quad , \quad A_2=\frac{T_ir_i^{\beta}-T_0r_0^{\beta}}{r_i^{\beta}-r_0^{\beta}}, \tag{4}$$

$$T(r)=-\frac{T_i-T_0}{-r_i^{-\beta}+r_0^{-\beta}}r^{-\beta}+\frac{T_ir_i^{\beta}-T_0r_0^{\beta}}{r_i^{\beta}-r_0^{\beta}} \tag{5}$$

where T_i and T_0 are the inner and outer temperatures at r_i and r_0 respectively.

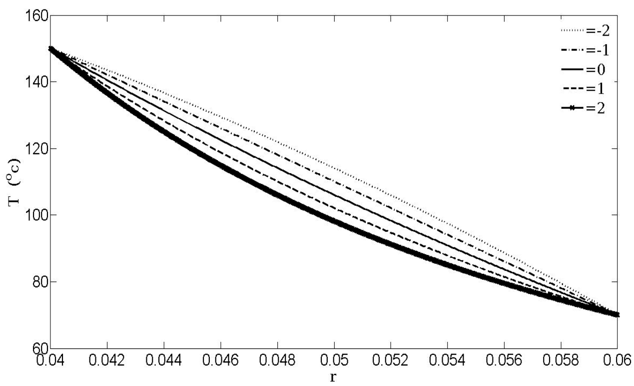

Fig. 2. Temperature distribution versus radius for five grading index.

3. Formulation Based on the FSDT Technique

In the FSDT, the assumption is that the planes normal to the mid-plane remain plane after deformation but not necessarily perpendicular to it after loading and the consequent deformations. In this case, shear strain and stress are considered. In the classical theory of shells, it is assumed that the planes normal to the mid-plane remain plane even after deformation occurs.

According to the selected coordinate system (r, x, θ) one can write:

$$r=R+z, \qquad -\frac{h}{2}\leq z\leq +\frac{h}{2} \tag{6}$$

where h and L are the shell thickness and length of the cylinder.

The general axisymmetric displacement field (U_r, W_x, V_θ) according to Mirsky-Hermann's first-order theory is expressed on the basis of axial and radial displacements, as follows:

$$\begin{aligned} U_r(x,\theta) &= U(x,\theta) + Z\phi_r(x,\theta) \\ W_x(x,\theta) &= W(x,\theta) + Z\phi_x(x,\theta) \\ V_\theta(x,\theta) &= V(x,\theta) + Z\phi_\theta(x,\theta) \end{aligned} \tag{7}$$

where ϕ_r, ϕ_x and ϕ_θ are the middle surface rotation components. Furthermore, U, W and V are the functions used to define the displacement field. The strain-displacement formulas in the cylindrical coordinate system are:

$$\begin{cases} \varepsilon_{rr} = \dfrac{\partial u}{\partial z} + z\dfrac{\partial \phi_z}{\partial z} + \dfrac{\partial z}{\partial z}\phi_z = \phi_z \\ \varepsilon_{\theta\theta} = \dfrac{1}{r}(u + z\phi_z) = \dfrac{u}{R+z} + z\dfrac{\phi_z}{R+z} \\ \varepsilon_{xx} = \dfrac{\partial w}{\partial x} + z\dfrac{\partial \phi_x}{\partial x} \\ \gamma_{r\theta} = \dfrac{1}{r}\dfrac{\partial u_z}{\partial \theta} + \dfrac{\partial v_\theta}{\partial z} - \dfrac{v_\theta}{r} = \phi_\theta - \dfrac{v}{r} - z\dfrac{\phi_\theta}{r} \\ \gamma_{rx} = \dfrac{\partial w_x}{\partial z} + \dfrac{\partial v_z}{\partial x} = \phi_x + \dfrac{\partial u}{\partial x} + z\dfrac{\partial \phi_z}{\partial x} \\ \gamma_{x\theta} = \dfrac{1}{r}\dfrac{\partial w_x}{\partial \theta} + \dfrac{\partial v_\theta}{\partial x} = \dfrac{\partial v}{\partial x} + z\dfrac{\partial \phi_\theta}{\partial x} \end{cases} \tag{8}$$

Stress-strain relations are written as follows:

$$\begin{cases} \sigma_{rr} = \dfrac{E}{(1+v)(1-2v)} \\ \qquad [(1-v)\varepsilon_{rr} + v(\varepsilon_{\theta\theta} + \varepsilon_{xx}) - (1+v)\alpha T] \\ \sigma_{\theta\theta} = \dfrac{E}{(1+v)(1-2v)} \\ \qquad [(1-v)\varepsilon_{\theta\theta} + v(\varepsilon_{rr} + \varepsilon_{xx}) - (1+v)\alpha T] \\ \sigma_{xx} = \dfrac{E}{(1+v)(1-2v)} \\ \qquad [(1-v)\varepsilon_{xx} + v(\varepsilon_{rr} + \varepsilon_{\theta\theta}) - (1+v)\alpha T] \\ \tau_{r\theta} = K\dfrac{E}{2(1+v)}\gamma_{r\theta}, \quad \tau_{rx} = K\dfrac{E}{2(1+v)}\gamma_{rx}, \\ \tau_{\theta x} = K\dfrac{E}{2(1+v)}\gamma_{\theta x} \end{cases} \tag{9}$$

According to the principle of virtual work, the variations of strain energy must be set equal to the variations of the external work as follows:

$$\delta U = \delta W \tag{10}$$

where δU is the variation of total strain energy of the elastic body and δW is the variation of total external work due to internal, external pressure, friction force, and centrifugal body force. The strain energy is then written as:

$$\begin{aligned} U = \iiint_v \overline{u}\,\mathrm{d}v &= \int_0^2 \pi \int_0^L \int_{-h/2}^{+h/2} \overline{u} r\,\mathrm{d}r\,\mathrm{d}x\,\mathrm{d}\theta \\ &= 2\pi \int_0^L \int_{-h/2}^{+h/2} \overline{u}(R+z)\,\mathrm{d}z\,\mathrm{d}x \end{aligned} \tag{11}$$

where:

$$\Rightarrow \overline{u} = \frac{1}{2}\,\frac{E}{(1+v)(1-2v)}$$

$$\begin{bmatrix} [(1-v)](\varepsilon_{rr}^2 + \varepsilon_{\theta\theta}^2 + \varepsilon_{xx}^2) + 2v(\varepsilon_{rr}\varepsilon_{\theta\theta} + \varepsilon_{rr}\varepsilon_{xx} + \varepsilon_{\theta\theta}\varepsilon_{xx}) \\ -(1+v)\alpha T(\varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{xx}) + \dfrac{K(1-2v)}{2}[\gamma_{r\theta}^2 + \gamma_{rx}^2 + \gamma_{\theta x}^2] \end{bmatrix} \tag{12}$$

The external work is the sum of works due to internal, external pressure (W_1), centrifugal body force (W_2) and friction force (W_3):

$$W = W_1 + W_2 + W_3 \tag{13}$$

where:

$$\begin{cases} W_1 = \displaystyle\int_0^L (d_1 u + d_2\phi_z)\,\mathrm{d}x \\ W_2 = \displaystyle\int_0^L (H_1 u + H_2\phi_z)\,\mathrm{d}x \\ W_3 = \displaystyle\int_0^L (I_1 v + I_2\phi_z)\,\mathrm{d}x \end{cases} \tag{14}$$

That:

$$\begin{aligned} d_1 &= 2\pi\left[P_i(x)\left(R - \frac{h}{2}\right) - P_0(x)\left(R + \frac{h}{2}\right)\right] \\ d_2 &= 2\pi\frac{h}{2}\left[-P_i(x)\left(R - \frac{h}{2}\right) - P_0(x)\left(R + \frac{h}{2}\right)\right] \\ H_1 &= 2\pi\rho_0\omega^2\left(R^2 h + \frac{R^2h^3}{2} + \frac{h^5}{80}\right) \\ H_2 &= 2\pi\rho_0\omega^2\left(\frac{R^2h^3}{3} + \frac{Rh^5}{40}\right) \\ I_1 &= -2\pi\mu P_0(x) \\ I_2 &= -2\pi\left(\frac{h}{2}\right)\mu P_0(x) \end{aligned} \tag{15}$$

Taking variation from energy relation we have:

$$\begin{aligned} U &= \int_0^L (U_s - U_T)\,\mathrm{d}x - \int_0^L (W_1 + W_2 + W_3)\,\mathrm{d}x \\ &= \int_0^L F(u, w, v, \phi_z, \phi_x, \phi_\theta)\,\mathrm{d}x \end{aligned} \tag{16}$$

Among the 6 variables in the relations obtained, Euler's equations are:

$$\frac{\partial F}{\partial q_i} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial\left(\dfrac{\partial q_i}{\partial x}\right)}\right) = 0, \tag{17}$$

$$q_i(i = 1,2,3,4,5,6) = u, \phi_r, w, \phi_x, v, \phi_\theta$$

where F is general potential function.

By applying the Euler equations and using equations written in appendix A system of differential equations governing the problem is obtained as follows:

$$[G_1]\frac{\mathrm{d}^2}{\mathrm{d}x^2}\{y\}+[G_2]\frac{\mathrm{d}}{\mathrm{d}x}\{y\}+[G_3]\{y\}=\{F\}$$
$$\{y\}=\{u,\phi_r,w,\phi_x,v,\phi_\theta\}^T \tag{18}$$

Matrix G_1, G_2, G_3 and F are written in appendix B.

Solving the differential equations, general and particular solutions are written as follows:

$$\{y\}=\{y_g\}+\{y_p\} \tag{19}$$

That general solution is represented as follows:

$$\{y_g\}=\{v\}_i e^{mx} \tag{20}$$

Finally, by substituting special values, the general solution is written as follow:

$$\{y_g\}=\sum_{i=1}^{10} c_i\{v\}_i e^{m_i x}+c_{11}x+c_{12} \tag{21}$$

$c_{11}x+c_{12}$ term is due to a pair of zero roots. Given the presence of mixed roots, these roots need to be transformed into a true form to continue solving [26]:

$$\lambda_{i,i+1}=a\pm bi \tag{22}$$

Then the special vectors derived from these roots will be in the following form:

$$\lambda_{i,i+1}=\Gamma\pm\Omega i \tag{23}$$

Particular solution consists of thermal and mechanical components as:

$$\{y\}_p=\{y\}_{p1}+\{y\}_{p2} \tag{24}$$

Finally, with determination of unknown coefficients the solution is obtained as:

$$\{y\}=\sum_{i=1}^{10} c_i\{v\}_i e^{m_i x}+c_{11}x+c_{12}+\{y\}_p \tag{25}$$

Substituting the solution into Eqs. (9), (10) the stresses and strains can be calculated where the formulation of effective stress and strain in terms of stress and strain components is as follows:

$$\sigma_e=\frac{1}{\sqrt{2}}\begin{bmatrix}(\sigma_{rr}-\sigma_{\theta\theta})^2+(\sigma_{rr}-\sigma_{xx})^2\\+(\sigma_{\theta\theta}-\sigma_{xx})+6\tau_{r\theta}^2+6\tau_{rx}^2+6\tau_{\theta x}^2\end{bmatrix}^{\frac{1}{2}} \tag{26}$$

$$\varepsilon_e=\frac{\sqrt{2}}{3}\begin{bmatrix}(\varepsilon_{rr}-\varepsilon_{\theta\theta})^2+(\varepsilon_{rr}-\varepsilon_{xx})^2\\+(\varepsilon_{\theta\theta}-\varepsilon_{xx})+6\varepsilon_{r\theta}^2+6\varepsilon_{rx}^2+6\varepsilon_{\theta x}^2\end{bmatrix}^{\frac{1}{2}} \tag{27}$$

4. Results and Discussion

In this section the numerical results for effective stress and strain components are presented in terms of grading index. Power law distribution for radial dependent properties of FGM [3] is written as follows:

$$P=P_0\left(\frac{r}{R_i}\right)^\beta \tag{28}$$

where P represents a property and the P_0 is the reference material property.

The reference properties, geometry and loading data used in this paper for rotating FG hollow cylinder are assumed to be:

Table 1
Basic material properties, geometry and loading data used in this paper.

E_0	220GPa
v	0.3
α_0	$1.2\times10^{-06}\,\frac{1}{°\mathrm{C}}$
R_i	0.04m
R_0	0.06m
P_i	80MPa
P_0	30MPa
T_i	150°C
T_0	70°C
l	1m

4.1. Comparison and Validation

Before presentation of full numerical results, a comparison with other results is required. For this aim, the numerical analysis based on Abaqus package was selected. Shown in Fig. 3 is comparison between present results using analytical method and corresponding results using the Abaqus software for a case study. This comparison indicates that the present results are in good agreement with results of numerical analysis. Type of element that was used is C3D8T and number of elements for Convergence were 50160.

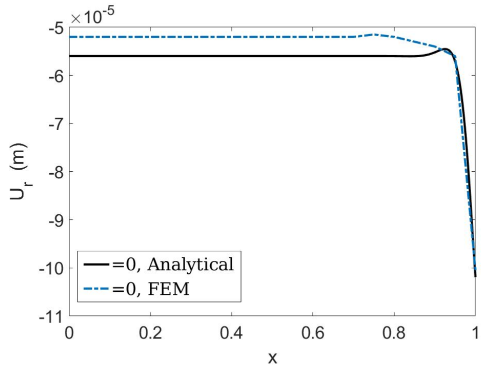

Fig. 3. Comparison between the present results using analytical method and corresponding results using the Abaqus software.

Effective stress distribution in radial direction is shown in Fig. 4. For negative values of grading index and for homogeneous material ($\beta = 0$) maximum values of effective stresses are located at the inner surface of the shell and their minimum values are located at the outer surface of the shell. However, there are no significant changes throughout thickness for positive values of grading index. In Fig. 5 shear strain versus radius is shown and the minimum absolute value of which belongs to a material identified by the grading index $\beta = +2$ and the maximum absolute value belongs to $\beta = -2$.

In Figs. 6 and 7 shear strains in longitudinal direction are shown. Except for the end condition there is no significant changes for different material properties. Figs. 8, 9, and 10 present radial, longitudinal and tangential displacements, respectively.

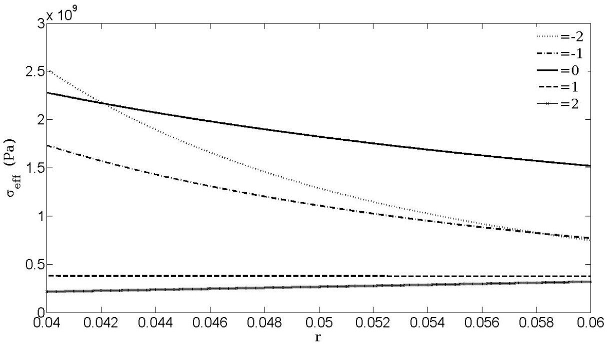

Fig. 4. Effective stress (σ_{eff}) versus radius.

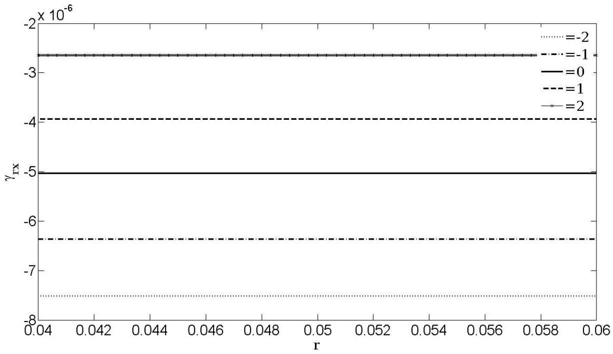

Fig. 5. Shear strain (γ_{rx}) versus radius.

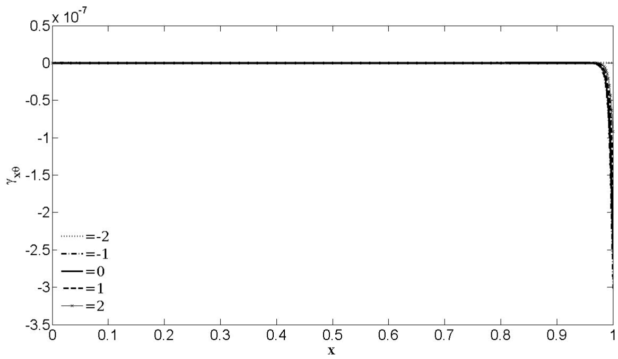

Fig. 6. Shear strain ($\gamma_{x\theta}$) versus length.

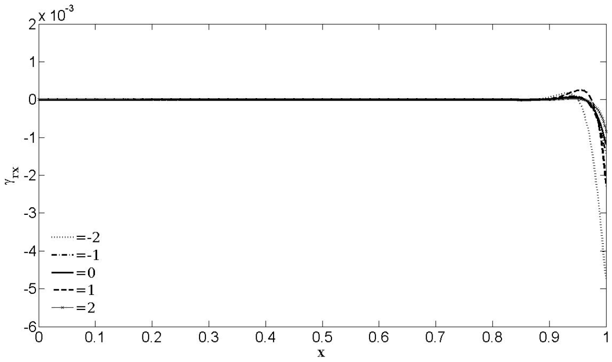

Fig. 7. Shear strain (γ_{rx}) versus length.

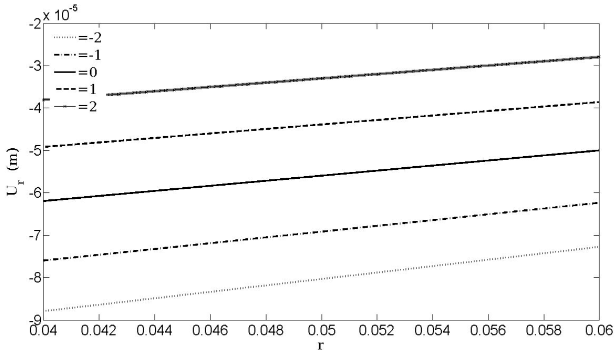

Fig. 8. Radial displacement (U_r) versus radius.

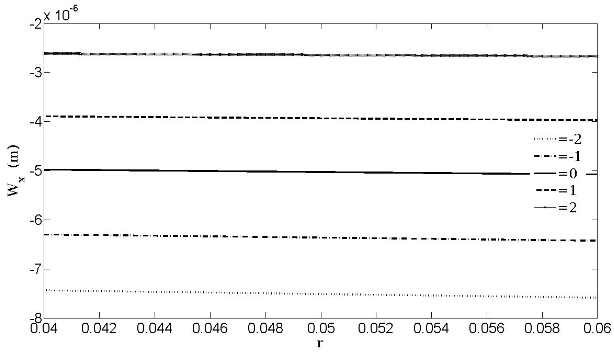

Fig. 9. Longitudinal displacement (W_x) versus radius.

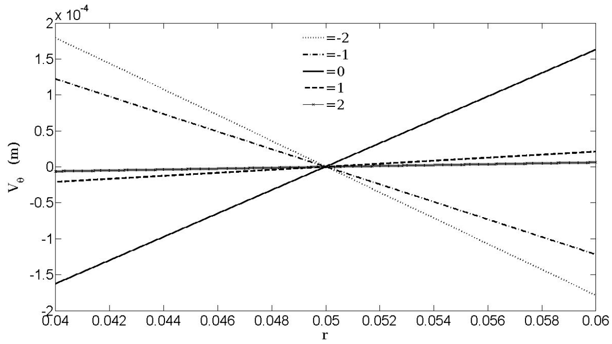

Fig. 10. Tangential displacement (V_θ) versus radius.

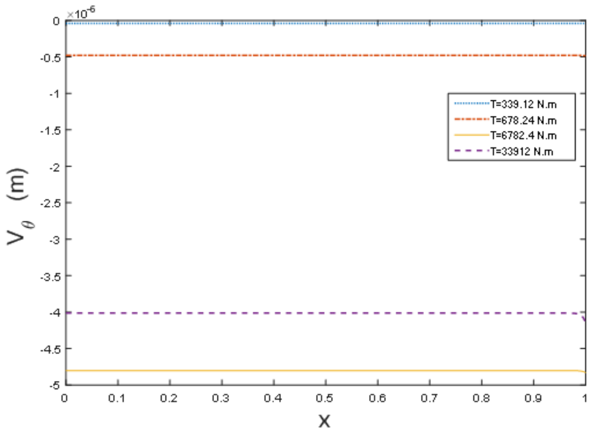

Fig. 11. Tangential displacement (V_θ) versus length for external torque for ($\beta = 0$).

5. Conclusions

In this work, formulation of 3D thermo-elastic analysis of an FG rotating cylinder subjected to internal/external pressure and shear stresses due to friction bed was performed using first-order shear deformation theory (FSDT). The mechanical properties except Poisson's ratio were variable along the radial direction according to a power law distribution. By using the boundary conditions, constant coefficients of the six differential equations were obtained. It is concluded that the grading index has a significant effect on stresses and strains.

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