Journal of Stress Analysis
Vol. 3, No. 2, Autumn − Winter 2018-19

Assessing Reliability of Bending of Concrete Beams Exposed to Freeze-thaw Conditions Based on Compressive Stress Limit Reduction

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Article info

Article history:
Received 23 December 2018
Received in revised form
16 March 2019
Accepted 17 March 2019

Keywords:
Reliability
Freeze-thaw cycles
Stress limit
Concrete
Stochastic model
Compression

Abstract

For existing reinforced concrete structures exposed to freeze-thaw conditions, there is an increasing engineering concernover their remaining safety. This paper presents a novel experimental-theoretical stochastic model for evaluating the reliability of concrete structures subjected to freeze-thaw conditions based on stress limit reduction. Reliability theory and experimental works provide the basis for the model development. Water cement ratio, air content, and number of freeze-thaw cycles are considered as the model variables. Compressive stress limit reduction in freeze-thaw conditions was treated as a stochastic variable. The effectiveness of the proposed model was evaluated using an example concrete structure element. The paper demonstrates that after, for example, 10 years experiencing FT cycles in a cold city; the reliability of the example concrete beam reduces to 52.5 percent for −10°C concrete freezing temperature. It was found that the results of the proposed method are accurate compared to the literature. It was also found that the results of the proposed method are in good agreement with those obtained based on concrete's non-destructive tests.

1. Introduction

For existing reinforced concrete structures exposed to freeze-thaw (FT) conditions, there is an increasing engineering concern in their remaining safety and serviceability, including the expected remaining life. Freezing pressure in the cement paste or concrete aggregates in each FT cycle results in local redistribution of moisture during the freezing period. This is followed by the absorption of moisture from outside the concrete or internal redistribution of moisture during the thawing period [1]. The freeze-thaw damage (deterioration) of concrete occurs when internally generated hydrostatic pressure during FT process is greater than the destruction produced by tensile strength of concrete [2]. Similar to what occurs in normal mechanical fatigue, each new cycle adds to cumulative internal damage [3, 4] resulting in the growth of microcracks caused by internal damage of concrete. With higher FT cycles, microcracks gradually turn into macrocracks leading to major concrete internal damage. Another type of concrete frost damage is surface scaling (peeling) [5]. Internal damage seems to be more severe than surface scaling as internal damage leads to a considerable loss in the mechanical properties of concrete [6].

The methods available to assess the frost damage of concrete tend to be both experimental, mainly non-destructive, [7] and theoretical [3, 8]. The available methods generally look at the loss of dynamic modulus of elasticity as an indicator expressing the internal damage of concrete.

Tang et al. provided a review on the recent studies

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http://dx.doi.org/10.22084/jrstan.2019.17989.1075
ISSN: 2588-2597

on the durability of concrete exposed to environmental attacks, including FT conditions [9]. They called for new studies concerning on more accurate service life estimate of concrete structures exposed to environmental attacks. Wawrzenczyk and Molendowska analyzed the damage process of concrete exposed to FT conditions stochastically based on the mass changes in specimens [10]. Ashra et al. developed a design approach for assessing the FT durability of concrete based on relative loss in dynamic modulus of elasticity using a sensitivity analysis and a probabilistic approach [11]. It was found that paste content of the concrete mixtures, air content of hardened concrete, and number of FT cycles are the most sensitive factors. In another study, Smith et al. defined FT failure as the likelihood that the hydraulic pressure caused by freezing process exceeds the tensile strength of the hardened cement paste [12]. FT data were used from literature to develop their probabilistic model using a lognormal distribution. It was concluded that air-void spacing, saturation state, freezing rate, and permeability of concrete significantly influence the concrete FT performance. Duan et al. developed a stochastic frost damage method based on micromechanical modelling, with two variables which were determined by a nonlinear regression analysis [13]. Based on conducting FT tests on seven high strength concrete mixtures, Bumanis et al. found coherence between compressive strength and ultrasonic pulse velocity results [14]. Fagerlund treated actual and critical moisture contents in concrete as stochastic variables to calculate the probability of frost damage using a triangular probability density function [15]. Penttala presented a nonlinear damage model based on three variables (water-cement ratio, air content, and curing time) and five coefficients which were determined by a regression analysis [7]. Nili et al. developed a theoretical model to predict the internal damage of concrete using mathematical modeling [8]. Jun et al. gave a frost damage model for initial and developing phases of concrete damage [16]. Similarly, Liu and Wang suggested a tri-linear model for internal deterioration of concrete [17]. Zhou and Sun looked at concrete freeze-thaw damage by combining stochastic damage theory with thermodynamics [2].

Despite of seminal works of above studies, they generally look at damage mechanisms of concrete through foc using on the loss of dynamic modulus of elasticity. In practice, however, structural engineers need to know stress limit reduction of concrete (for example compressive stress) caused by FT attacks in order to assess the reliability of a given structure. Few studies have proposed a method which is appropriate for predicting concrete compressive stress limit and the associated reliability in FT conditions. This is mainly because of difficulty in conducting destructive tests of concrete (typically for assessing compressive stress of concrete) compared to non-destructive testes (typically for assessing dynamic module of elasticity) under FT cycles. Shang and Song argued the lack of any report on the compressive stress limit of concrete subjected to FT cycles [18]. Nevertheless, most of existing frost damage methods are deterministic and therefore they fail to consider the stochastic nature of concrete [2]. FT processes involve great uncertainty [13]. Presumably, it is more practical to use a stochastic method rather than a deterministic one for evaluating the compressive stress limit of concrete. Stochastic methods have popularly been used for assessing the durability of concrete exposed to carbonation or chloride ingress [e.g., 20, 21]. However, few stochastic methods have been proposed to assess the reliability of concrete exposed to FT conditions [13, 15].

In such light, this paper proposes a novel experimental-theoretical stochastic model for evaluating the reliability of concrete structures subjected to FT cycles through focusing on the compressive stress limit of concrete. The model features both stochastic and stress limit. Mathematical formulas, reliability theory, and experimental works provide the basis for the model development. Water cement ratio and air content of concrete were treated as random variables. The present model offers an experimental-theoretical basis for exploring the stochastic aspect of concrete compressive stress limit against FT attacks.

The rest of the paper is structured as follows. The problem formulation is first given. Then the proposed model is provided. This is followed by demonstrating the application and accuracy of the proposed model.

2. Material and Methods

2.1. Problem Formulation

The basic reliability problem considers one load effect S, also referred to as structural response or internal actions, restrained by one limit (resistance) L, also known as an acceptable stress capacity for structural response [21]. Typically, S is calculated from the applied load via a conventional structural analysis procedure. Both S and L should be calculated in the same units. Owing to deterioration (frost damage) of concrete in FT conditions, the value of R decreases by time leading to a reduction in the reliability of the concrete structure. In evaluating the reliability of a concrete structure, a criterion should be established. In reliability theory, this criterion can be expressed by a limit state function, as follows [22],

$$G(S, L) = S - L \tag{1}$$

From Eq. (1), the structural reliability, denoted by R, can be determined by [22],

$$R = P[G(S - L) < 0] = P[S < L] \tag{2}$$

where $P[\ ]$ denotes the probability of an event. Following reliability theory [e.g., 24], S and L are described by a known probability density function, $f_S(u)$ and $f_L(v)$, respectively, where u and v are random variables. It is followed directly that when S and L are random variables Eq. (2) can be expressed as,

$$R = \iint_D f_{SL}(u,v)\,\mathrm{d}u\,\mathrm{d}v \tag{3}$$

where $f_{SL}(u,v)$ is the joint probability density function of S and L; and D is the domain that represents $S < L$.

When S and L are independent, Eq. (3) can be obtained by (Melchers and Beck, 2018),

$$R = \int_{-\infty}^{+\infty} F_L(z) f_S(z)\,\mathrm{d}z \tag{4}$$

where $F()$ is a cumulative distribution function; $f()$ is a probability density function; and z is a random variable.

As a special case when S is a normal random variable and L has a deterministic value, the analytical solution of Eq. (4) is possible and can be obtained by the reliability index β [21]. But this might not be always the case in reliability problems.

To apply Eqs. (3) or (4) to the problem of reliability of concrete exposed to FT cycles, the main effort lies in developing stochastic models of load effect S and stress limit L. This is explained in the following section. For illustration, the reliability of a structure element rather than a whole structure is discussed.

2.2. Proposed Method

Solving Eqs. (3) or (4) might be followed in two parts, namely part I and part II, shown in Fig. 1. In part I, first variables influencing stress limit (L) need to be defined. This is followed by determining a deterministic model of L, for example based on laboratory observations. Then, probability density functions of variables affecting L need to be established. Next, the probabilistic model of S is calculated.

In part II, the probabilistic model of load effect S is determined. For this purpose, the structural element, its load and variables effecting S need to be defined. This is followed by determining the deterministic model of S. Then by establishing probability density functions of variables of S, the probabilistic model of S can be obtained. If both L and S possess known probability density functions, Eqs. (3) or (4) can be solved and reliability of the structural element can then be obtained. In order to investigate the existence of aprobability density function for L and S, the Kolmogorov-Smironov qualitative test was used [23]. This test might be used to match a set of quantitative data to different probability distribution functions such as, normal, Poisson, exponential, and uniform.

In the case when L and S do not possess a probabilistic density function (PDF) other forms of L and S, such as reverse and logarithmic are tried for data matching. If other forms of L and S possess a probability density fuction, Eq. (3) can be solved for those forms (for example reverse). Otherwise, Eq. (3) cannot be solved in closed (analytical) form and instead a numerical technique (such as, Monte Carlo simulation) needs to be adopted to generate values of L and S randomly (for example 10000 times). Then such values are compared pairwise. The ratio of number of pairs with $S < L$ to the total number of comparisons are reported as the reliability value, as shown in Fig. 1.

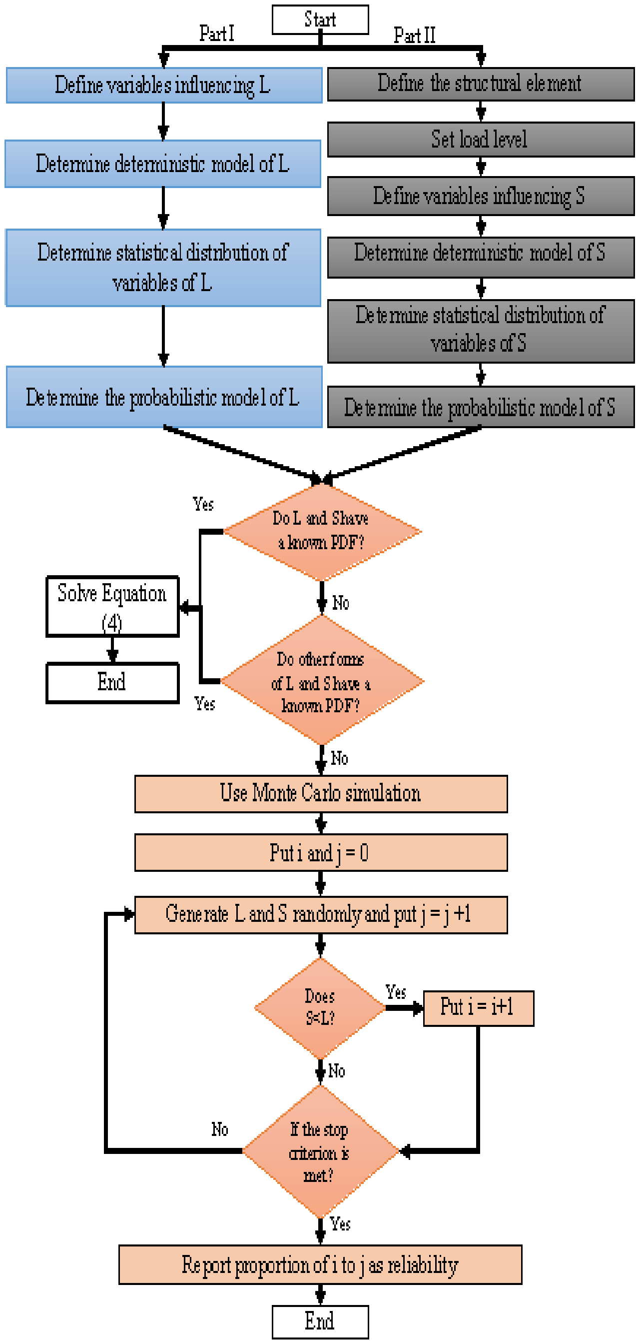

Fig. 1. Flow chart for calculating reliability of a concrete structural element.

Table 1
Compressive stress limit of concrete (MPa).

W/c	Air content(%)	At zero cycle	After 45 cycles	After 100 cycles	After 150 cycles	After 200 cycles	After 250 cycles
0.5	3	35.8	23.6	14.0	0	0	0
	4.5	32.6	28.2	27.5	26.5	9.0	0
	6	29.3	25.7	25.3	24.7	7.7	0
	7.5	27.2	23.9	23.7	23.2	7.6	0
0.6	3	30.0	18.1	8.1	0	0	0
	4.5	25.0	20.7	19.7	18.3	5.5	0
	6	22.2	18.6	18.3	17.6	5.7	0
	7.5	19.5	17.5	17.3	16.8	5.5	0
0.7	3	21.8	13.0	2.9	0	0	0
	4.5	18.2	13.9	13.1	11.7	3.9	0
	6	16.2	13.5	12.7	12.2	3.9	0
	7.5	14.8	12.8	12.6	12.2	3.7	0
0.8	3	16.1	8.5	0	0	0	0
	4.5	13.5	8.6	6.8	5.5	1.7	0
	6	11.6	8.8	8.1	7.5	2.5	0
	7.5	11.4	9.4	9.2	8.5	2.7	0

2.3. Reliability Assessment

In practice, generally a 60 percent loss in dynamic modulus of elasticity of concrete exposed to FT cycles is considered as an acceptable limit for reliability of concrete against FT conditions [24]. However, this limitmight not be practical for structural analysis as it is regarded as an indication showing the durability of concrete exposed to FT cycles. Concrete compressive stress limit might be more practical for reliability assessment, though other stress types such as shear stress might be considered. To provide a stochastic model for concrete compressive stress limit exposed to FT cycles it is first essential to develop a deterministic model of it. Unfortunately, such models have not well developed in the literature perhaps due to destructive nature of compressive tests coupled with time consumption nature of FT tests. An experimental test was conducted to develop a deterministic model for compressive stress limit of concrete against FT cycles as discussed below.

3. Results and Discussion

3.1. Experimental Tests

For developing a deterministic model for concrete compressive stress limit in FT conditions, a series of experimental tests with three variables, namely water to cement ratios (0.5, 0.6, 0.7, 0.8), concrete air contents (3, 4.5, 6, 7.5 percent), and number of FT cycles were considered. Sixteen mix designs wereconsidered for concrete samples. Type I ordinary Portland cement was used. Coarse aggregates were crushed stone (with maximum aggregate size of 9.5mm; with grading matched to curve no. eight of ASTM C33 [25]) and fine aggregates were limestone with fineness modulus of 2.66, grading based on ASTM C33 [25]. For each compressive test, two specimens were considered and the averages of the results were reported. The specimens were prepared in cubic forms ($10 * 10 * 10$ centimeters), due to the space limit of FT apparatus. The FT tests were conducted based on ASTM C-666B. All specimens were removed from molds 24h after casting and then were cured under water in lab conditions ($20 \pm 3°C$) for 14 days before they were exposed to FT tests. According to ASTM C-666B, each FT cycle lasted three hours, in which 20% of the time, specimens were in thawing, and in 80% of the time, specimens were frozen. Concrete specimens were exposed to 250 FT cycles and compressive tests were conducted after 0, 45, 100, 150, 200 and 250 cycles. Specimens were capped before compressive tests according to ASTM C617 for obtaining smooth and level surfaces [26]. The compressive test machine was electro-hydraulic with a capacity of 2000kN. The average loading rate was 0.38MPa/sec. Air content wasmeasured based on ASTM C231 [27]. Overall, 192 compressive tests were conducted. The slump of concrete mixes were measured based on ASTM 143 [28]. The slump values varied between 70mm to 110mm. Table 1 presents the corresponding results. The detailed information on the FT tests and the compressive strength loss was reported earlier [29].

3.2. Deterministic Model of L

Using the data of Table 1, and conducting a regression analysis, with $R^2 = 0.93$, a deterministic model for compressive stress limit (L in Eq. (2) and Fig. 1) can be obtained by,

$$L = \frac{15.97A^{0.98}}{\left(\frac{W}{c}\right)^{0.51} N^{0.10}} - 10A \tag{5}$$

where A is the concrete air content (taking values between three and 7.5 percent); is water cement ratio (taking values between 0.5 and 0.8); and N is number of FT cycles (taking values between zero and 250).

3.3. Probabilistic Model of L

To develop a probabilistic model for L it is required to account uncertainty information in the variables involved. For this purpose, a probabilistic densityfunctionneeds to be considered for each involved variable to treat it as a stochastic variable. Different probability distribution functions might be considered. Following Li a normal distribution function was adopted in this research [30]. Table 2 sets out estimates for uncertainties in variables of Eq. (5) based on experimental data, Table 1, and the (limited) information in the literature [30].

Table 2
Uncertainty estimates for variables of Eq. (5).

Variable	Mean	Standard deviation
$\frac{W}{C}$	0.65	0.05
$A\,(\%)$	5.25	0.76

By using Eq. (5) and Table 2, the probabilistic model of L can be obtained using a simulation technique. In this paper, Monte Carlo simulation is adopted. Such algorithmwas popularly used in the literature for example to simulate chloride diffusion in concrete exposed to de-icing salts [31], to study mesoscale fracture behavior of concrete with random aggregates [32] and to assess the strength of concrete columns [33]. A Monte Carlo simulation algorithm typically utilizesa random sampling approach to obtain numerical results for the given dependent variable [34]. Such algorithm first, based on a uniform distribution function, generates a set of random numbers between zero and one. Then it uses the cumulative distribution function of each considered probabilistic independent variable and random numbers generated to find a value for the variable. By having values of all independent variables, next the algorithm calculates the value of the dependent variable based on a given formula. This procedure is repeated many times, for example 10000 times, to generate a set of data for the dependent variable [34].

In the current study, the simulation was conducted in R which isa language and environment for statistical computing. For this purpose a code was written. Fig. 2 shows the simulation results after 45 FT cycles. The index in this Figure represents the iteration numbers. Similar results can be presented for different FT cycles.

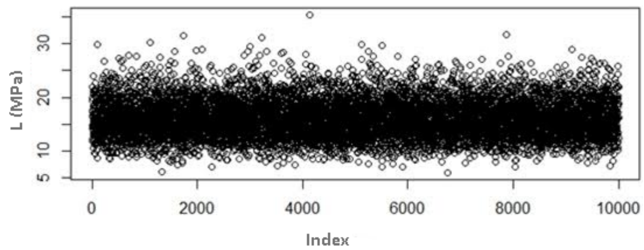

Fig. 2. Stochastic compressive stress of concrete after 45 FT cycles.

To match the generated L data to a probability distribution function such as, normal, Poisson, exponential, and uniform, the Kolmogorov-Smironov qualitative test was conducted [23]. This test might be used to compare a set of data with a reference probability distribution via quantifying a distance between the empirical distribution function of the data and the cumulative distribution function of the reference distribution. Given a set of data, the null hypothesis is that the set of data fits to a reference probability distribution and the alternative hypothesis is that the set of data failsto fit to the reference probability distribution. A p-value less than 0.05 rejects the alternative hypothesis meaning the set of data comes from the reference probability distribution [23].

Table 3 presents p-values obtained from such test for different statistical distribution functionsin different FT cycles. A value for p bigger than 0.05, indicates the existence of a probability distribution function. Table 3 shows that natural logarithm of L at 45 and 100 cycles and square root of L at 150 and 200 cycles only match a probability distribution function (normal). Table 4 presents mean and standard deviation results for the probabilistic forms of compressive stress limit L in different FT cycles.

3.4. Example Application of Proposed Method

To show the application of the proposed method it is required to consider a concrete element. For this purpose a double-cantilever beam, shown in Fig. 3, was considered.

Table 3
Amount of p-values for L.

Distribution forms	Form \ Cycles	45	100	150	200
Normal	L	0.012	0.02	0.031	0.03
	$\log(L)$	0.0184	0.0159	0.0428	0.023
	$1/L$	0.0249	0.0378	0.0215	0.028
	$\sqrt{L}$	0.0438	0.0251	0.0836	0.094
	$\ln(L)$	0.139	0.096	0.0119	0.010
Exponential	L	0.0001	0.0001	0.0000	0.0000
Poisson	L	0.0000	0.0000	0.0000	0.0000
Uniform	L	0.0001	0.0001	0.0000	0.0000

Table 4
Mean and standard deviation results for L (MPa).

Freeze-thaw cycles	45	100	150	200
Function form	$\ln(L)$	$\ln(L)$	$\sqrt{L}$	$\sqrt{L}$
Mean	2.74	2.4	2.94	1.59
Standard deviation	0.21	0.26	0.45	0.22

Table 5
Cross-sectional and other properties, and their uncertainty estimates for example beam.

Variable	Units	Mean	Standard deviation
L	mm	5000	10
h	mm	400	10
b	mm	250	10
d	mm	350	10
A_s (reinforcement area)	mm^2	339.3	5
f_y (yield strength of steel bars)	MPa	400	0
W	N/mm	38.4	0

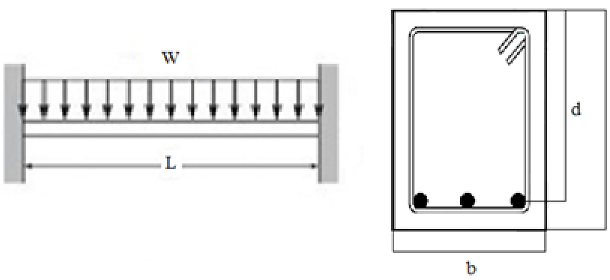

Fig. 3. Example beam, dimension layout, and cross-section.

The cross-sectional propertiesmay be treated as deterministic. It is known that matters such as dimensions and concrete cover are subject tovariability in practice [35]. In this paper, following Melchers et al., (2008) [35] the cross-sectional properties were treated as stochastic variables with normal distribution. Table 5 shows the cross-sectional and other properties, and their uncertainty estimates for the example beam based onthe (limited) information in the literature [30].

3.5. Deterministic Model of S

To determine a deterministic model of the load effect S for the considered beam, its mechanical properties, such as bending moment and shear force need to be considered. For illustration, the maximum positive moment (Mn) at the middle of the span is considered and calculated according to the conventional structural analysis,

$$M_n = \frac{WL^2}{24} \tag{6}$$

where W is the uniform load intensity. Assuming that the Whitney Stress Block is well-suited here (ACI 318-14) [36], the nominal momentcapacity of the beam can be written as,

$$M_n = A_s f_y \left(d - \frac{A_s f_y}{2 \times 0.85 \times b \times S} \right) \tag{7}$$

In Eqs. (6) and (7) the coefficients of load increase and capacity reduction are setone due to the stochastic nature of the problem. From Eq. (7), the deterministic model of S can be obtained,

$$S = \frac{A_s f_y}{1.7b}\left(d - \frac{WL^2}{24A_s f_y}\right)^{-1} \tag{8}$$

By using Monte Carlo simulation, Eq. (8) and data of Table 5, the stochastic model of S can be calculated by simulation in the R platform. To investigate the existence of any probability distribution function for simulation results, the Kolmogorov-Smironov qualitative test was used. The corresponding results are shown in Table 6. These results demonstrate lack of any probability distribution function matching ($p < 0.05$) for different forms of S.

Table 6
Amount of p-values for S.

Form	S	$\log(S)$	$1/S$	$\sqrt{S}$	$\ln(S)$
p-value	0.0248	0.0049	0.0119	0.0397	0.0412

3.6. Reliability Calculation

Due to the lack of any probability density function for Sit is not possible to calculate reliability of the example concrete beam from Eq. (3). Thus, as discussed early, a numerical method needs to be used, (see Fig. 1). For this purpose Monte Carlo simulation was used and a code was written in R. The corresponding results are shown in Fig. 4. This figure shows the reliability of the example beam in different FT cycles. According to the results, for example, the reliability of the beam decreases to 52.5 percent after 100 cycles. This shows how vulnerable concrete is to FT attacks. Therefore concrete structures should be protected against FT in cold areas.

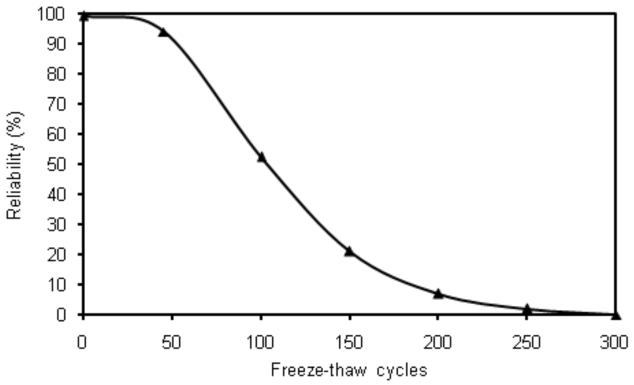

Fig. 4. Reliability of example beamin different FT cycles.

Typically, reliability of concrete structures needs to be calculated based on the age of structures rather than number of FT cycles. To present the reliability based on the structure age, the number of FT cycles per year in the region that the structure is located needs to be identified. For illustration, a city, called Hamedan, with cold climate conditions, located in western Iran, was considered. The weather temperature data of this city, provided by the local weather forecasting authority, was studied in two recent years (2015 and 2016), due to difficulty in data collection. Unfortunately, there is no standard or universal agreement on the concrete freezing and thawing temperatures in nature. In this study negative 10°C was considered as concrete freezing temperature, and zero°C was considered as thawing one. Based on this assumption, Table 7 shows the number of FT cycles in different months in Hamedan in 2015 and 2016. The average of FT cycles for this city is 10 cycles per year for these two years. For illustration, the average FT cycles in this city was assumed to be 10 cycles per year. It is acknowledged that by enlarging the weather temperature data over recent years a more accurate predication might be obtained.

By using Table 7, the reliability of the example concrete beam in the Hamedan climate can be calculated. The corresponding results are shown in Fig. 5. This figure demonstrates that after, for example, 10 years experiencing FT cycles in Hamedan the example beam's reliability considerably decreasesto 52.5 percent. Fig. 5 also compares the reliability of the beam for different concrete freezing temperatures. As it is expected, this figure shows that the results for −8°C are more conservative than those for −10°C. For example according to Fig. 5, reliability of the beam after six years becomes 82 percent for −10°C concrete freezing temperature while for −8°C the reliability decreases to 52.5 percent. This demonstrates how sensitive is the reliability results to concrete freezing temperature. Further studies need to be conducted to address freezing and thawing temperatures of concrete.

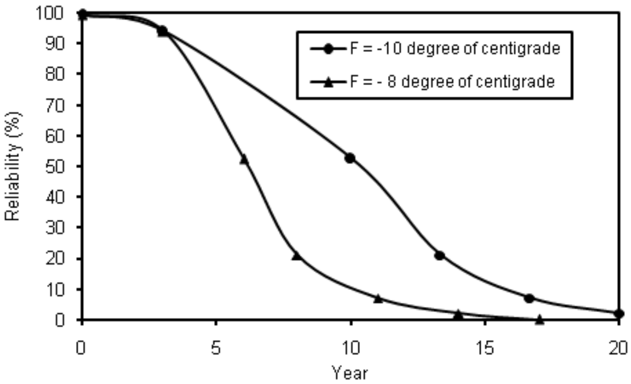

Fig. 5. Reliability of example beams for different concrete freezing temperature (F).

Table 7
Number of FT cyclesfor Hamadan in 2015 and 2016.

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total
2015	3	0	0	0	0	0	0	0	0	0	3	2	7
2016	6	0	0	0	0	0	0	0	0	0	5	3	14

3.7. Validation

There are almost no experimental or theoretical results concerning reliability of concrete exposed to FT conditions to compare the proposed method. The past studies generally focused on non-destructive methods such as reduction of the dynamic modulus of elasticity of concrete, and their approaches are generally deterministic rather than probabilistic. Few stochastic methods have been proposed to assess the reliability of concrete exposed to FT conditions [13]. Following Nili et al. [8], the probabilistic model presented by Duan et al. [13] is used to compare the results. Duan et al. [13] developed a stochastic frost damage method based on micromechanical modeling without discussing the stress limit reduction of concrete structures, which is crucial for the structural engineer or asset manager to predict the remaining life of the structure. In order to provide a basis for comparing the results of the paper with those of the Duan et al. study [13], this paper-compared reduction in compressive stress limit of concrete, denoted by D_{Model}, with those of the dynamic module of elasticity given in Duan et al. [13]. Fig. 6 comparing the model predictions with those of Duan et al. [13] illustrates a good agreement between these two models. The average value of D_{Model}/D_{Duan} et al. is 1.04, reflecting a good accuracy for the proposed model. To measure the significant of this, a t-test analysis was conducted. Using Fig. 6 data, Table 8 details the result of the t-test.

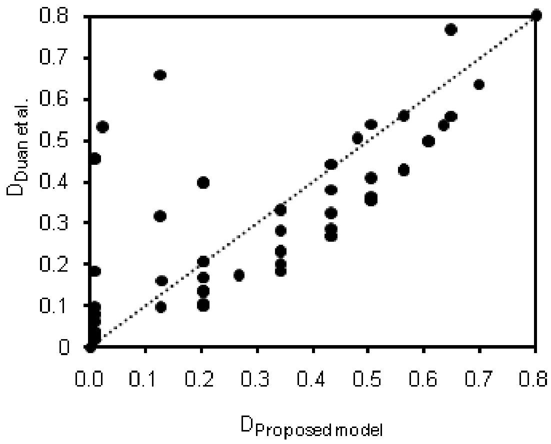

Fig. 6. Comparison of results with Duan et al. [13].

It is noted that the smaller the t-value is, the less likely the difference is significant. A critical t-value is the minimum t-value required to have $p < 0.05$. If the t-value is less than or equal to the critical t-value, then the difference between the two data sets are not statistically significant [37]. According to Table 8, the t-value is less than the t-critical value with a p-value higher than 0.05. This demonstrates that the difference between the results produced by the proposed method and those of Duan et al. [13] are not statistically significant [37]. This suggests that the proposed method can produce very similar, results to those generated by Duan et al. [13]. This provides support for the paper findings. The results also approve that reduction in the compressive stress limit of concrete is in a good agreement with the reduction in the dynamic modulus of elasticity. This confirms that measuring the amount of loss in the dynamic modulus of elasticity is a good indicator for showing frost damage and consequently reduction in reliability of concrete.

Table 8
The t-test result for comparing results produced by the proposed model and that of Duan et al. [13].

t-value	−0.026
p-value one-tail	0.489
t-critical-value (one-tail)	1.656
p-value two-tail	0.979
t-critical-value (two-tail)	1.977

To further validate the results of this paper the experimental data (tests), which was obtained from Zhang [38], presented in Table 9 was used. This data set was earlierused by Nili et al. [8] to validate their model. Fig. 7 comparing the model predictions with those of Zhang [38] demonstrates a good agreement between these two models. The average value of D_{Model}/D_{Zhang} is 0.97, showing a good accuracy for the proposed model.

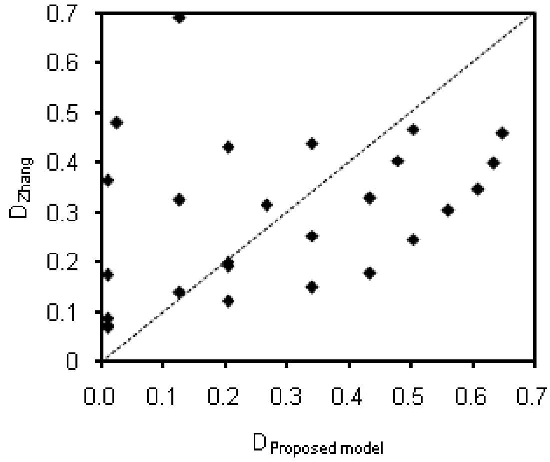

Fig. 7. Comparison of results with Zhang [38].

Table 9
Values of D for different FT cycles.

W/c	N	D_{Zhang}	$D_{\text{Proposed model}}$
0.4	25	0.069	0.010
	50	0.123	0.205
	75	0.149	0.341
	100	0.179	0.435
	125	0.246	0.505
	150	0.303	0.562
	175	0.345	0.609
	190	0.398	0.634
	200	0.458	0.649
0.45	25	0.072	0.010
	40	0.14	0.127
	50	0.192	0.205
	60	0.315	0.267
	75	0.439	0.341
0.5	25	0.086	0.010
	50	0.2	0.205
	75	0.253	0.341
	100	0.329	0.435
	115	0.404	0.479
	125	0.467	0.505
	175	0.345	0.609
	125	0.467	0.505
0.4	25	0.174	0.010
	40	0.326	0.127
	50	0.363	0.205
0.6	25	0.363	0.010
	30	0.479	0.025
	40	0.691	0.127

4. Conclusions

This paper proposed a novel stochastic model for evaluating the reliability of concrete subjected to FT cycles. The focus was on concrete compressive stress limit reduction. Mathematical formulas, reliability theory, and experimental works provided the basis for the model development. Water cement ratio and air content weretreated as random variables. The results show that natural logarithm of compressive stress limit at 45 and 100 cycles and square root of it at 150 and 200 cycles only match the normal probability distribution. The paper also shows that the values of mean (standard deviation) of natural logarithm of compressive stress limit at 45 and 100 cycles are 2.74MPa (0.21MPa) and 2.4MPa (0.26MPa), respectively while those of square root of compressive stress limit at 150 and 200 cycles are 2.94MPa (0.45MPa) and 1.59MPa (0.22MPa), respectively. A case example was analyzed to show the application of the proposed method and to demonstrate its capabilities in predicting the reliability of a concrete structure element exposed to FT cycles. It was found that:

1. The results of the proposed method are accurate compared to the literature.

2. The reliability of concrete structures depends considerably on the freezing temperature of concrete. For example, the paper demonstrates that aftersix years experiencing FT cycles in Hamedan, the reliability of the considered beam decreasesto 82 percent for $-10°$C concrete freezing temperature, while for $-8°$C concrete freezing temperature the reliability considerably decreasesto 53 percent.

3. Predicting the reliability of concrete based on reduction in compressive stress limit is in good agreement with that of non-destructive methods.

4. The proposed method can serve as an accurate tool for structural engineers and asset managers in making decisions with regard to repairs, strengthening, and/or rehabilitation of frost affected bending strength of concrete beams.

5. It is acknowledged that the development on the stochastic model of concrete is based on a beam elementand its validity for other elements or structures should be investigated separately. This paper could be regarded as a basic framework for further development, and might be expected to be applicable to different concrete element.

The empirical studies in this paper were conducted based on 192 samples, with normal concrete. Further empirical testing could be carried out on different types of concrete to enlarge the sample and provide further persuasive support for the paper's findings.

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