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Developing of Chebyshev Collocation Spectral Method for Analysis of Multiple Structural Mechanics

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Abstract

In this work, the spectral collocation method based on Chebyshev polynomials is developed and utilized for analysis of static, free vibration, and dynamic behavior of one and two-dimensional solid structures. The main objective of the work is to introduce the spectral collocation method with Chebyshev polynomials as a powerful numerical method for solid mechanic analysis. To show the advantage and effortlessness of this method, one and two-dimensional solid structures as case studies were considered and the spectral collocation method was directly applied to the analysis and the governing equation was solved. Moreover, the homogeneous material properties and functionally graded material properties were analyzed to show the capability of the introduced method for solving the more complicated equations of motion. The results obtained for each case were compared with analytical and numerical results presented in the literature and some results were also compared with ANSYS. The results showed that the presented method has very good accuracy and efficiency to solve structural-mechanical properties.

Nomenclature

$T_n(x)$	Chebyshev polynomials of the first kind	F_x	Body forces in x direction
a_i	Unknown Chebyshev coefficients	F_y	Body forces in y direction
$\theta(x,y)$	Temperature	u_x	Displacement fields in x direction
L_x	Length of plate in x direction	u_y	Displacement fields in y direction
L_y	Length of plate y direction	λ, μ	Lame´'s constants
E_R	Young's modulus in the right direction of nanobeam	E_L	Young's modulus in the left direction of nanobeam
ρ	Mass density		

1. Introduction

Due to the complexity of differential equations governing the solid mechanic problems (Navier equations), in most cases there is no way to find analytical solutions for these kinds of differential equations. The numerical solution is an alternative way to get the desired results in such situations. So some numerical methods are

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widely developed and used in this branch of engineering analysis. The most developed numerical methods are the Finite Element Method (FEM), Finite Difference Method (FDM), Mesh free Method, and some other methods that are not used extensively in comparison with the three mentioned methods.

From a general point of view, the numerical methods in a solid mechanic are categorized into two classes, weak and strong forms. Methods that applied to the weakened form of differential equations named weak forms and methods that directly imposed on the differential equations of problems belong to the strong form methods. In the strong form, it is necessary that the approximation of field variables has a strong degree of consistency in the problem domain whereas in the weak form a weaker consistency is needed. For example, for solving second order differential equations with a strong form method, the approximation function must be derivative at least twice, but in the weak form method, only one derivability of approximation function is enough. The weak form methods are more accurate and stable, and because of this reason they are developed more than strong form methods, but weak forms have complicated algorithms and heavy calculations, in contrast, strong form methods have simple algorithm and calculations but they are in most situations unstable and inaccurate, especially in problems with derived type boundary conditions. One of the most famous numerical strong form methods is the collocation method. In this method first, an approximation or interpolation method is made to approximate the field variables in the problem domain and then the system of an algebraic equation is generated by satisfying the governing differential equation at some points named collocation points. Chebyshev polynomials are most used as trial functions in spectral methods. The spectral method is a method for representing the dynamic solution in the form of a series of solutions at different frequencies, this method is mostly used in the fluid dynamic analysis [1-3]. One of the important advantages of spectral methods is their high accuracy in analysis with numbers of node discretization that are remarkably less than other numerical methods, but they have a serious disadvantage that may be the reason that why they are not developed and used extensively in many branches of engineering such as a solid mechanics. In two and three dimensions, spectral methods applicable to domains that their boundaries are parallel to coordinate axis (such as rectangle and cub) and more generally to domains that can be mapped to a rectangle or cube. Although some applications are presented that use them in some sub-domains such as spectral element method and even the same manner is used for fluid dynamic analysis in nonrectangular domains or mono-domain and multi-domain solutions of fluid equations, but these applications do not have the simplicity of direct spectral methods [4,5]. Some studies deal with collocation methods are as follows: Zhang et al. [6] proposed a finite point method, least-squares collocation meshless method, for solving Poisson equation and also some static structural analysis. They concluded that the proposed collocation method was performed better than the direct collocation method whereas the total number of unknowns in their method equals those in direct collocation method. In another work, Liu et al. [7] used the radial point interpolation collocation method (RPICM) based on Hermite-type interpolation for the solution of 2-D solid mechanic analysis. Lee and Yoon [8] used a Generalized Diffuse Derivative Approximation (GDDA) in combination with a point collocation method for analysis of elasticity and crack problems.

Free vibration analysis of Timoshenko beams by Discrete Singular Convolution (DSC) method was performed by Civalek and Kiracioglu [9]. Clamped, pinned, and sliding boundary conditions and their combinations were taken into account in their work and they concluded that DSC method is very effective for the study of vibration problems of Timoshenko beam.

Large deflection static analysis of rectangular plates on two-parameter elastic foundations was investigated by Civalek and Yavas [10]. They studied geometrically nonlinear static analysis of thin rectangular plates on Winkler-Pasternak elastic foundation and the nonlinear partial differential equations obtained from Von Karman's large deflection plate theory. Finally, they solved the obtained nonlinear partial differential equations by using the DSC method. Vibration analysis of Functionally Graded (FG) cylindrical shells with power-law index using discrete singular convolution technique was investigated by Mercan et al. [11].The constitutive relations were based on the Love's first approximation shell theory and the material properties of cylindrical shell were considered to be graded in the thickness direction according to a volume fraction power law indexes.

Static analysis of functionally graded piezoelectric plates under electro-thermo-mechanical loading using meshfree method based on radial point interpolation method (RPIM) was investigated by Nourmohammadi and Behjat [12]. The First-order Shear Deformation Plate Theory (FSDT) was used to model the behavior of the plate and also power law distribution through the thickness was considered for all of mechanical, thermal, and piezoelectric properties in their work. Sheikhi Azqandi et al. [13] presented a novel hybrid method by considering the strengths and weaknesses of the two methods of the Direct Sensitivity Method (DSM) and the complex variables method (CVM) and combining them to calculate shape sensitivity in solid mechanics.

Furthermore, some studies have been conducted around the spectral methods and Chebyshev polynomials that some of them are as follows: Zhou et al. [14] employed Chebyshev polynomials multiplied by a boundary function as admissible functions in Ritz method for free vibration analysis of rectangular plates with various thicknesses. They concluded that the proposed technique yields very accurate natural frequencies and mode shapes of rectangular plates with arbitrary boundary conditions. Celik [15] presented a solution for magneto hydrodynamic flow in a rectangular duct by the Chebyshev collocation method. Chebyshev polynomials were used for approximation of field variables and then the collocation was applied to equations at a reasonable number of collocation points. Carcione [16] used a 2D Chebyshev differential operator for solving the elastic wave equation with nonperiodic boundary conditions. Moreover, he successfully used the technique to domain decomposition and applied proper boundary conditions on domain interfaces. Ehrenstein and Peyret [17] carried out a solution of Navier-Stokes equations with double-diffusive convection with the Chebyshev collocation method, they concluded that the collocation method possesses some advantages over the Tau method such as better accuracy and stability, easier solution of variable coefficients equations. Wu et al. [18] studied the application of Chebyshev spectral method for the numerical solution of time-dependent variably saturated Darcian flow problems. They showed that Chebyshev spectral method has higher computational efficiency than the traditional finite difference method in their problem because finite difference method requires a high mesh density to improve accuracy.

Huang et al. [19] used Chebyshev spectral method as a new approach for vibration analysis of in-plane functionally graded plates with variable thickness. Both the material properties and the thickness which vary in the plane of the plate were approximated by high-order Chebyshev expansions. They showed that the results obtained from the Chebyshev spectral method have a good convergence and agree with those in literature.

Free axisymmetric vibrations of composite circular sandwich plates with isotropic core and orthotropic facings were studied using first-order shear deformation theory by Rani and Lal [20]. They used the Hamilton's principle to derive the governing differential equations and finally they applied Chebyshev collocation technique to obtain the frequency equations for the plate with clamped or simply supported or free edge conditions.

Free axisymmetric vibrations of composite annular sandwich plates with thick isotropic core and orthotropic facings by using Reddy's higher-order shear deformation theory were studied by Guru and Jain [21]. They used Chebyshev collocation technique to determine the frequency equations and then solved them using hybrid bisection-secant method to obtain the frequency values for first three modes of clamped-clamped, clamped-simply supported, and clamped-free plates. Alihemmati et al. [22, 23] developed Chebyshev collocation method for generalized thermoelasticity problems of one and two-dimensioanl finite domains. They solved the highly coupled thermoelasticity equations based on Lord-Shulman, Green-Lindsay and Green-Naghdi theories by Chebyshev collocation method and concluded that the used Chebyshev collocation method besides its simplicity has very good convergence and accuracy in generalized thermoelasticity problems.

Chih-Hsun and Ming-Hw [24] solved the governing differential equations of a laminated anisotropic plate by utilizing the Chebyshev collocation method. The solution of the problem was assumed to be a set of Chebyshev polynomials with some unknown constants and several examples were given to highlight the effectiveness of this method.

Gumgum et al. [25] investigated two-dimensional heat equation by using Chebyshev collocation method. The method converts the two-dimensional heat equation to a matrix equation, which corresponds to a system of linear algebraic equations.

In the literature, there are many different solution methods for the governing equations in macro and micro/nano dimensions [26-34]. In recent years, several articles were presented to show the capabilities of different methods [35, 36]. But the point to be noted is that each of these methods is applicable to certain problems and there are fewer comprehensive solution methods that are suitable for all engineering problems. Anyway, Chebyshev collocation method is a straightforward and easily implemented method and beside its simplicity shows very good accuracy and stability in other works performed in the literature. It can be suitable to apply this method to various kinds of solid mechanics analysis as well as various geometries to prove the capability of the Chebyshev collocation method to use successfully in structural mechanics. So the present work is an attempt to develop an application for the Chebyshev collocation method to solve governing equations for static, free vibration, and dynamic analysis of some solid structures. Additionally, some complicated cases as functionally graded material properties and nano-structural analysis with non-classical elasticity theories are considered. Here, we have tried to analyze the problems that have more applications in mechanical engineering and also tried to solve some case studies in macro-micro dimensions to show the capability of this method more than before. The results are compared with other results found in other literature and also with ANSYS.

2. Methodology

2.1. Chebyshev Polynomials

The Chebyshev polynomials of the first kind are the polynomials of n-degree defined as follows [1]:

$$T_n(x) = \cos(n\cos^{-1}x) \qquad -1 \le x \le 1,\ n = 0, 1, 2, \cdots \tag{1}$$

so:

$$\begin{aligned} T_0(x) &= 1 \\ T_1(x) &= x \\ T_2(x) &= 2x^2 - 1 \\ T_3(x) &= 4x^3 - 3x \\ T_4(x) &= 8x^4 - 8x^2 + 1 \end{aligned} \tag{2}$$

One of the most important advantages of the Chebyshev polynomial that makes it much flexible to use in numerical simulation of physical phenomena is that these polynomials can be achieved by the recurrence as:

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x) \tag{3}$$

It means that by having the first and second Chebyshev polynomial, T_0, and T_1, other polynomials can easily derived by Eq. (3).

The integral and derivative of any order of these polynomials can also be derived in terms of the Chebyshev polynomials as follows:

$$\frac{T'_{n+1}(x)}{(n+1)} - \frac{T'_{n-1}(x)}{n-1} = 2T_n(x) \qquad n \ge 2 \tag{4}$$

and

$$\int T_n(x)dx = \frac{1}{2}\left[\frac{T_{n+1}(x)}{(n+1)} - \frac{T_{n-1}(x)}{n-1}\right] + C \quad n \ge 2 \tag{5}$$

Eqs. (4) and (5) show that the derivates and integrals of any order of Chebyshev polynomials can be derived easily in terms of the polynomials themselves, that these rules beside the recurrence rule of Eq. (3) are very useful in numerical and algorithmic solutions.

2.2. Interpolation with Chebyshev Polynomials

A function $y(x)$ in the interval of $[-1, 1]$ can be interpolated by Chebyshev polynomial as:

$$\begin{aligned} y(x) &= a_0T_0 + a_1T_1 + a_2T_2 + \cdots + a_nT_n \\ &= \sum_{i=1}^{n} a_iT_i(x) \end{aligned} \tag{6}$$

in which a_i are the coefficient of n^{th} degree of Chebyshev polynomial that approximate $y(x)$ in the interval $[-1, 1]$ and pass through $n+1$ nodes (x_i, y_i).

Since $y(x)$ passes through $n+1$ nodes (x_i, y_i), a set of $n+1$ equations are generated as follows:

$$\begin{aligned} &y(x_0) = y_0 \quad \Rightarrow \\ &a_0T_0(x_0) + a_1T_1(x_0) + a_2T_2(x_0) + \cdots + a_nT_n(x_0) = y_0 \\ &y(x_1) = y_1 \quad \Rightarrow \\ &a_0T_0(x_1) + a_1T_1(x_1) + a_2T_2(x_1) + \cdots + a_nT_n(x_1) = y_1 \\ &\vdots \\ &y(x_n) = y_n \quad \Rightarrow \\ &a_0T_0(x_n) + a_1T_1(x_n) + a_2T_2(x_n) + \cdots + a_nT_n(x_n) = y_n \end{aligned} \tag{7}$$

and in matrix form:

$$\begin{bmatrix} T_0(x_0) & T_1(x_0) & \cdots & T_n(x_0) \\ T_0(x_1) & T_1(x_1) & \cdots & T_n(x_1) \\ \vdots & \vdots & \vdots & \vdots \\ T_0(x_n) & T_1(x_n) & \cdots & T_n(x_n) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_n \end{bmatrix} \tag{8}$$

And finally the coefficients (a_i) can be derived by solving this system of algebraic equations.

It should be noted that for a polynomial of n^{th} degree, using of first n of Chebyshev polynomials is sufficient for interpolation and this interpolation gives exact solution.

2.3. Spectral Collocation with Chebyshev Polynomials

Now the idea presented in section 3.1 is employed to interpolate the field variable in differential equations. Consider a differential equation as follows:

$$\begin{aligned} &A(x)\frac{d^2u}{dx^2} + B(x)\frac{du}{dx} + C(x) = F(x) \qquad -1 \le x \le 1 \\ &u(-1) = u_1 \qquad \left.\frac{du}{dx}\right|_{x=1} = k \end{aligned} \tag{9}$$

where $A(x)$, $B(x)$, $C(x)$, and $F(x)$ are arbitrary functions of x, and u_1 and k are scalars. For the case of the domain intervals aside from $[-1,1]$, without any change in the generality of the method, the problem domain can be mapped into $[-1,1]$ by a simple mapping.

For applying the Chebyshev collocation spectral method to this differential equation first the field variable u is interpolated by Chebyshev polynomials as:

$$u=\sum_{i=0}^{n} a_i T_i \tag{10}$$

a_i are unknown Chebyshev coefficients that need to be evaluated and T_i are Chebyshev polynomials that approximate u in the whole domain of problem with n^{th} degree polynomial. This is the basic difference between the spectral method and other numerical methods such as finite element and finite difference methods that interpolate/approximate the variables in some subdomains. After interpolation, the collocation is applied to Eq. (9) at Chebyshev nodes as:

$$A(x_j)\sum_{i=0}^{n} a_i \left.\frac{d^2T_i}{dx^2}\right|_{x=x_j} + B(x_j)\sum_{i=0}^{n} a_i \left.\frac{dT_i}{dx}\right|_{x=x_j} = F(x_j) \qquad j=2,3,\cdots,n \tag{11}$$

$$\sum_{i=0}^{n} a_i T_i\Big|_{x=-1} = u_1$$

$$\sum_{i=0}^{n} a_i \left.\frac{dT_i}{dx}\right|_{x=1} = k$$

x_j are collocation points, as for this work where the Chebyshev nodes are used as collocation points. The numbers of collocation points are restricted by a maximum number of Chebyshev polynomials and also the types of boundary conditions, for example in Eq. (11) at each boundary nods we have one equation for boundary condition, it means that $n+1$ collocation points should be considered, in some situations such as beam equations at boundary points we may have more than one equation that decreases the total numbers of collocation points. By applying Eq. (11) in collocation points a system of $n+1$ set of the equation is generated as Eq. (12).

By solving the above system of equations, the unknown coefficient will be determined and so u can be calculated at any arbitrary point that is the goal of this problem with Eq. (10). In the following part, some examples of the performance of the introduced method are presented.

3. Results and Discussion

In this section, two-dimensioanl steady-state heat conduction is considered the first case study and after analyzing this case, this method was applied for analyzing the other complex problems in solid mechanics. The application of this introduced method in solid mechanics is the main goal of the present work. In special cases, the method is applied to static, dynamic, and free vibration analysis of one- and two-dimensional macro/microstructures. Furthermore, to show the flexibility of the present method in a more complicated problem the same analysis of Functionally Graded Material (FGM) cases was performed.

$$\underbrace{\begin{bmatrix} T_0(-1) & T_1(-1) & \cdots & T_n(-1) \\ A(x_2)\left.\frac{d^2T_0}{dx^2}\right|_{x=x_2}+ & A(x_2)\left.\frac{d^2T_1}{dx^2}\right|_{x=x_2}+ & \cdots & A(x_2)\left.\frac{d^2T_n}{dx^2}\right|_{x=x_2}+ \\ B(x_2)\left.\frac{dT_0}{dx}\right|_{x=x_2}+ & B(x_2)\left.\frac{dT_1}{dx}\right|_{x=x_2}+ & \cdots & B(x_2)\left.\frac{dT_n}{dx}\right|_{x=x_2}+ \\ A(x_3)\left.\frac{d^2T_0}{dx^2}\right|_{x=x_3}+ & A(x_3)\left.\frac{d^2T_1}{dx^2}\right|_{x=x_3}+ & \cdots & A(x_3)\left.\frac{d^2T_n}{dx^2}\right|_{x=x_3}+ \\ B(x_3)\left.\frac{dT_0}{dx}\right|_{x=x_3}+ & B(x_3)\left.\frac{dT_1}{dx}\right|_{x=x_3}+ & \cdots & B(x_3)\left.\frac{dT_n}{dx}\right|_{x=x_3}+ \\ \vdots & \vdots & \cdots & \vdots \\ A(x_{n-1})\left.\frac{d^2T_0}{dx^2}\right|_{x=x_{n-1}}+ & A(x_{n-1})\left.\frac{d^2T_1}{dx^2}\right|_{x=x_{n-1}}+ & \cdots & A(x_{n-1})\left.\frac{d^2T_n}{dx^2}\right|_{x=x_{n-1}}+ \\ B(x_{n-1})\left.\frac{dT_0}{dx}\right|_{x=x_{n-1}}+ & B(x_{n-1})\left.\frac{dT_1}{dx}\right|_{x=x_{n-1}}+ & \cdots & B(x_{n-1})\left.\frac{dT_n}{dx}\right|_{x=x_{n-1}}+ \\ \left.\frac{dT_0}{dx}\right|_{x=1}+ & \left.\frac{dT_1}{dx}\right|_{x=1}+ & \cdots & \left.\frac{dT_n}{dx}\right|_{x=1}+ \end{bmatrix}}_{\mathbf{K}} \underbrace{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{n-1} \\ a_n \end{bmatrix}}_{\mathbf{a}} = \underbrace{\begin{bmatrix} u_1 \\ F(x_2) \\ F(x_3) \\ \vdots \\ F(x_{n-1}) \\ k \end{bmatrix}}_{\mathbf{F}} \tag{12}$$

3.1. Case study 1: Heat Conduction Equation

At first, to show the ability of the method in other field equations, a two-dimensional steady-state heat conduction problem for a homogenous rectangular plate was considered and solved by this method. The heat conduction equation and boundary conditions are as follows [37]:

$$\begin{aligned} &\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}=0\\ &\theta(0,y)=0\\ &\theta(L_x,y)=\theta_0\\ &\frac{\partial\theta(x,0)}{\partial y}=0\\ &\theta(x,L_y)=0 \end{aligned} \tag{13}$$

where θ denotes the temperature and L_x and L_y are the length of plate in x and y directions. For solution, first the temperature field is approximated in problem domain with Chebyshev polynomials as follows:

$$\theta=\sum_{i=0}^{n}\sum_{j=0}^{m}a_{ij}T_i(x)T_j(y) \tag{14}$$

The derivatives of θ are derived as:

$$\begin{aligned} &\frac{\partial\theta}{\partial y}=\sum_{i=0}^{n}\sum_{j=0}^{m}a_{ij}T_i(x)\frac{\partial T_j(y)}{\partial y}\\ &\frac{\partial^2\theta}{\partial y^2}=\sum_{i=0}^{n}\sum_{j=0}^{m}a_{ij}T_i(x)\frac{\partial^2 T_j(y)}{\partial y^2}\\ &\frac{\partial^2\theta}{\partial x^2}=\sum_{i=0}^{n}\sum_{j=0}^{m}a_{ij}\frac{\partial^2 T_i(x)}{\partial x^2}T_j(y) \end{aligned} \tag{15}$$

And then the collocation is applied to equation and boundary conditions as:

$$\begin{aligned} &\sum_{i=0}^{n}\sum_{j=0}^{m}a_{ij}T_i(x)\frac{\partial^2 T_j(y)}{\partial y^2}\\ &+\sum_{i=0}^{n}\sum_{j=0}^{m}a_{ij}\frac{\partial^2 T_i(x)}{\partial x^2}T_j(y)=0\\ &i=1,2,\cdots,n-1,\ j=1,2,\cdots,m-1\\ &\sum_{j=0}^{m}a_{0j}T_i(0)T_j(y)=0\\ &\sum_{i=0}^{m}a_{nj}T_i(L_x)T_j(y)=\theta_0\\ &\sum_{i=0}^{n}a_{i0}T_i(x)\frac{\partial T_j(0)}{\partial y}=0\\ &T\sum_{i=0}^{n}a_{im}T_i(x)T_m(L_y)=0 \end{aligned} \tag{16}$$

By arranging all nodal equations, the set of algebraic equations similar Eq. (12) is generated as follows:

$$\mathbf{Ka}=\mathbf{F} \tag{17}$$

By solving this equation for unknown coefficients, the temperature at any point of the plate is computed using Eq. (14).

It must be said that two-dimensional steady-state heat conduction is selected here for comparison and verification of the results. The results of the presented method for the heat conduction equation are obtained and compared with the analytical solution in Ref. [37]. Moreover, to show the convergence and stability of the results, the temperatures were computed using various collocation point sets (n,m), and presented in Table 1. n and m are the numbers of collocation points in x and y direction, respectively. In this example geometrical dimensions are as below:

$$L_x=1$$
$$L_y=2$$

Table 1 shows that the Chebyshev collocation method which is a very accurate and has very rapid convergence in this analysis. Additionally, by increasing the collocation points, the results remain stable and converge more and more to the exact solution.

Table 1
Convergence and stability of temperature in a homogenous plate.

Points location	$(0.1,0.3)$	$(0.5,0.5)$	$(0.5,1.4)$	$(0.7,1.6)$	$(0.2,1)$
$(n,m)=(3,6)$	9.8236	49.2857	40.9702	56.6553	17.9755
$(n,m)=(6,12)$	9.8941	49.3930	40.3818	52.9455	18.4417
$(n,m)=(8,16)$	9.8923	49.4037	40.4196	52.8603	18.4354
$(n,m)=(10,20)$	9.8915	49.4076	40.4056	52.8431	18.4342
Analytical method [37]	9.8918	49.4034	40.4057	52.8253	18.4349

It should be mentioned that generally, the collocation methods have the problem of instability in their analysis especially in problems containing the derivate type boundary conditions, but in Chebyshev collocation method, as can be observed from Table 1, have no instability. This conclusion is confirmed in all examples solved in the present work.

3.2. Case study 2: Homogeneous Plate in Plain Strain Condition

Fig. 1 shows a homogenous plate in plane strain condition and its geometrical parameters in a Cartesian coordinate system.

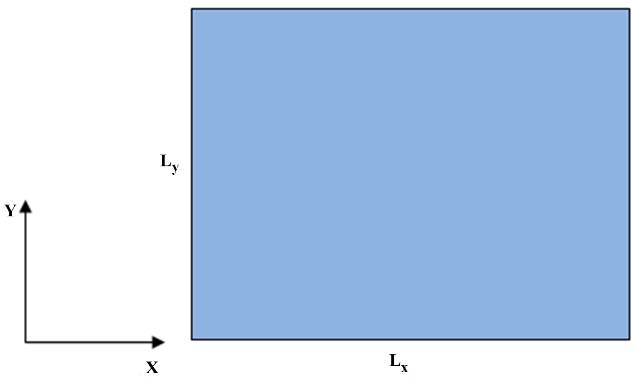

Fig. 1. Geometry and coordinates of homogenous plain strain plate.

For a two-dimensional plane strain problem, one can write Navier's or Lame´'s equations as follows [38]:

$$
\begin{aligned}
&\mu\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}\right)+(\lambda+\mu)\frac{\partial}{\partial x}\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}\right)\\
&\qquad +F_x=\rho\frac{\partial^2 u_x}{\partial t^2}\\
&\mu\left(\frac{\partial^2 u_y}{\partial x^2}+\frac{\partial^2 u_y}{\partial y^2}\right)+(\lambda+\mu)\frac{\partial}{\partial y}\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}\right)\\
&\qquad +F_y=\rho\frac{\partial^2 u_y}{\partial t^2}
\end{aligned}
\tag{18}
$$

where λ, μ, ρ, F_x, and F_y are Lame´'s constants, mass density, and body forces in x and y direction, respectively. The u_x and u_y are displacement fields in x and y direction, respectively. The displacement field is approximated with Chebyshev polynomial as follows:

$$
\begin{aligned}
u_x&=\sum_{i=0}^{n}\sum_{j=0}^{m}a_{ij}T_i(x)T_j(y)\\
u_y&=\sum_{i=0}^{n}\sum_{j=0}^{m}b_{ij}T_i(x)T_j(y)
\end{aligned}
\tag{19}
$$

and

$$
\begin{aligned}
\frac{\partial^2 u_x}{\partial x^2}&=\sum_{i=0}^{n}\sum_{j=0}^{m}a_{ij}\frac{\partial^2 T_i(x)}{\partial x^2}T_j(y),\\
\frac{\partial^2 u_x}{\partial x\partial y}&=\sum_{i=0}^{n}\sum_{j=0}^{m}a_{ij}\frac{\partial T_i(x)}{\partial x}\frac{\partial T_j(y)}{\partial y}\\
\frac{\partial^2 u_y}{\partial y^2}&=\sum_{i=0}^{n}\sum_{j=0}^{m}b_{ij}T_i\frac{\partial^2 T_j(y)}{\partial y^2},\\
\frac{\partial^2 u_y}{\partial x\partial y}&=\sum_{i=0}^{n}\sum_{j=0}^{m}b_{ij}\frac{\partial T_i(x)}{\partial x}\frac{\partial T_j(y)}{\partial y}\\
\frac{\partial^2 u_x}{\partial y^2}&=\sum_{i=0}^{n}\sum_{j=0}^{m}a_{ij}T_i(x)\frac{\partial^2 T_j(y)}{\partial y^2},\\
\frac{\partial^2 u_y}{\partial x^2}&=\sum_{i=0}^{n}\sum_{j=0}^{m}b_{ij}\frac{\partial^2 T_i(x)}{\partial x^2}T_j(y)
\end{aligned}
\tag{20}
$$

For dynamic analysis (time-dependent) coefficient, a_{ij} and b_{ij} are considered as functions of time, so

$$
\begin{aligned}
\frac{\partial^2 u_x}{\partial t^2}&=\sum_{i=0}^{n}\sum_{j=0}^{m}\ddot{a}_{ij}T_i(x)T_j(y),\\
\frac{\partial^2 u_y}{\partial t^2}&=\sum_{i=0}^{n}\sum_{j=0}^{m}\ddot{b}_{ij}T_i(x)T_j(y)
\end{aligned}
\tag{21}
$$

For essential and traction boundary conditions, the respective equations are as following: Essential B.C. example:

$$
u_x\Big|_{x=L_x}=0 \tag{22}
$$

Then

$$
\sum_{i=0}^{n}\sum_{j=0}^{m}a_{ij}T_i(L)T_j(y)=0\ \Rightarrow\ T_i(L)=0 \tag{23}
$$

Traction B.C. example:

$$
\text{at}\ \ y=L_y,\ \ \sigma_y=P(x),\ \ \sigma_{xy}=0 \tag{24}
$$

then:

$$
\begin{aligned}
&\Bigg[(2\mu+\lambda)\sum_{i=0}^{n}\sum_{j=0}^{m}a_{ij}\frac{\partial T_i(x)}{\partial x}T_j(y)\\
&\quad+\lambda\sum_{i=0}^{n}\sum_{j=0}^{m}b_{ij}T_i(x)\frac{\partial T_j(y)}{\partial y}\Bigg]_{y=L_y}=P(x)\\
&\Bigg[\sum_{i=0}^{n}\sum_{j=0}^{m}a_{ij}T_i(x)\frac{\partial T_j(x)}{\partial y}\\
&\quad+\sum_{i=0}^{n}\sum_{j=0}^{m}b_{ij}\frac{\partial T_i(x)}{\partial x}T_j(y)\Bigg]_{y=L_y}=0
\end{aligned}
\tag{25}
$$

By substituting Eqs. (20) and (21) in Eq. (19) and by satisfying them at collocation points (x_i, y_i), the system of dynamic equations is generated as follow:

$$\mathbf{M}\ddot{\mathbf{d}} + \mathbf{K}\mathbf{d} = \mathbf{F} \tag{26}$$

where,

$$\mathbf{d} = \begin{bmatrix} a_{ij} \\ b_{ij} \end{bmatrix}, \ddot{\mathbf{d}} = \begin{bmatrix} \ddot{a}_{ij} \\ \ddot{b}_{ij} \end{bmatrix}$$

Eq. (26) is the system of dynamic differential equations that should be solved to determine the values of a_{ij} and b_{ij} at any time. This set of differential equations can be solved with any initial value numerical methods, in this work, Newmark method was used.

For static analysis the right-hand sides of Eq. (18) are zero and so the mass matrix $\mathbf{M}$ in Eq. (26) vanishes and Eq. (26) is reduced to:

$$\mathbf{K}\mathbf{d} = \mathbf{F} \tag{27}$$

So a_{ij} and b_{ij} are derived by solving these simple algebraic equations. In both dynamic and static solutions by having the values of coefficients $a_1, a_2, \cdots, a_n$ and $b_1, b_2, \cdots, b_n$ and putting them in Eq. (19), the values of displacement, strains and stresses at any location (any time in dynamic cases) can be computed using Eq. (19) and Hook's low and also strain- displacement relations. In free vibration analysis cases Eq. (26) is reduced to:

$$\mathbf{M}\ddot{\mathbf{d}} + \mathbf{K}\mathbf{d} = \mathbf{0} \tag{28}$$

The natural frequencies are obtained by solving the eigenvalues of Eq. (28). As the first example of two-dimensional structural analysis, static analysis of a square domain in-plane strain condition is considered. The problem is solved by the presented Chebyshev collocation method and also the same problem is simulated in FEM by ANSYS software. Material properties and geometrical dimensions are as follows:

$$E = 207\text{GPa} \quad \nu = 0.3$$

$$L_x = L_y = 2(m)$$

And boundary conditions are:

$$\begin{aligned} &\text{At } x = -1 && \Rightarrow \ u_x = u_y = 0 \\ &\text{At } x = 1 && \Rightarrow \ \sigma_x = 1\text{MPa}, \ \sigma_{xy} = 0 \\ &\text{At } y = -1, 1 && \Rightarrow \ \sigma_x = \sigma_{xy} = 0 \end{aligned}$$

The displacements and stresses at some points of plate obtained by present method are shown in Tables 2 and 3, the same results obtained by ANSYS are also presented. This two tables show a very good accuracy of the used Chebyshev collocation methods for this static analysis.

3.3. *Case study 3: FG Cylinder in Axisymmetric Condition*

To show the ability of this method, an FG cylinder in the axisymmetric condition was analyzed. For the FGM axisymmetric model, λ and μ are function of radial coordinate (r), so the Navier's equations are not as same as Eq. (18), because the derivates of λ and μ with respect to coordinates are not zero and should be considered in equilibrium equations. The equilibrium equations for the axisymmetric condition are:

$$\begin{aligned} &\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{(\sigma_r - \sigma_\theta)}{r} + F_r = \rho a_r \\ &\frac{\partial \tau_{rz}}{\partial z} + \frac{\partial \sigma_z}{\partial z} + \frac{1}{r}\tau_{rz} + F_z = \rho a_z \end{aligned} \tag{29}$$

Table 2
Displacements for a square domain at some arbitrary points.

Displacement (μm)		$(-0.5, 0.5)$	$(0, -0.5)$	$(0, 0.7)$	$(0.5, -0.5)$	$(0.6, 0)$	$(0.7, 1)$	$(1, -1)$
u_x	Present method	1.9570	4.1564	4.2314	6.3672	6.7566	7.3216	8.6407
	ANSYS	1.9558	4.1543	4.2299	6.3659	6.7551	7.3218	8.6306
u_y	Present method	6.8235	9.2939	−1.3010	9.6965	0.0000	−1.9217	1.9521
	ANSYS	6.8081	9.2824	−1.3008	9.6882	0.0000	−1.9221	1.9500

Table 3
Stresses fora square domain at some arbitrary points.

Stresses (MPa)		$(-0.5, 0.5)$	$(0, -0.5)$	$(0.1, 0.8)$	$(0.5, -0.5)$	$(0.6, 0)$	$(0.6, 0.9)$	$(0.9, -0.8)$
σ_x	Present method	1.0198	1.0094	0.9740	1.0030	1.0084	0.9890	0.9982
	ANSYS	1.0198	1.0109	0.9747	1.0021	1.0081	0.9900	0.9992
σ_y	Present method	0.0919	0.0070	−0.0015	−0.0060	−0.0118	−0.0018	−0.0038
	ANSYS	0.0919	0.0080	−0.0003	−0.0065	−0.0124	−0.0005	−0.0028
σ_{xy}	Present method	−0.0436	−0.0110	0.1472	−0.0135	0	0.0063	−0.0022
	ANSYS	−0.0445	−0.0111	0.1440	−0.0138	0	0.0054	-0.0025

Hook's low with variable material property is written as:

$$\sigma_{ij} = \lambda(r)(\varepsilon_r + \varepsilon_\theta + \varepsilon_z)\delta_{ij} + \mu(r)\varepsilon_{ij} \tag{30}$$

Using strain-displacement relations in cylindrical coordinate and Eq. (30), then substituting both of them in the equilibrium Eq. (29), where λ and μ are function of r, the equilibrium is rewritten as:

$$\begin{aligned}
&(\lambda+2\mu)\frac{\partial^2 u_r}{\partial r^2} + \mu\frac{\partial^2 u_r}{\partial z^2} + (\lambda+\mu)\frac{\partial^2 u_z}{\partial r\partial z} + \left(\frac{\partial\lambda}{\partial r}\right.\\
&\left.+\frac{\lambda+2\mu}{r} + 2\frac{\partial\mu}{\partial r}\right)\frac{\partial u_r}{\partial r} + \frac{\partial\lambda}{\partial r}\frac{\partial u_z}{\partial z} + \left(\frac{1}{r}\frac{\partial\lambda}{\partial r}\right.\\
&\left.-\frac{\lambda+2\mu}{r^2}\right)u_r = \rho\frac{\partial^2 u_r}{\partial t^2}
\end{aligned} \tag{31}$$

$$\begin{aligned}
&\mu\frac{\partial^2 u_z}{\partial r^2} + (\lambda+2\mu)\frac{\partial^2 u_z}{\partial z^2} + (\lambda+\mu)\frac{\partial^2 u_r}{\partial r\partial z} + \left(\frac{\partial\mu}{\partial r}\right.\\
&\left.+\frac{\lambda+\mu}{r}\right)\frac{\partial u_r}{\partial z} + \left(\frac{\partial\mu}{\partial r} + \frac{\mu}{r}\right)\frac{\partial u_z}{\partial r} = \rho\frac{\partial^2 u_z}{\partial t^2}
\end{aligned}$$

The u_r and u_z are displacement fields in radial and axial direction, respectively, and are approximated with Chebyshev polynomial as follows:

$$\begin{aligned}
u_r &= \sum_{i=0}^{n}\sum_{j=0}^{m} a_{ij}T_i(r)T_j(z)\\
u_z &= \sum_{i=0}^{n}\sum_{j=0}^{m} b_{ij}T_i(r)T_j(z)
\end{aligned} \tag{32}$$

and

$$\begin{aligned}
\frac{\partial u_r}{\partial r} &= \sum_{i=0}^{n}\sum_{j=0}^{m} a_{ij}\frac{\partial T_i(r)}{\partial r}T_j(z),\\
\frac{\partial^2 u_r}{\partial r^2} &= \sum_{i=0}^{n}\sum_{j=0}^{m} a_{ij}\frac{\partial^2 T_i(r)}{\partial r^2}T_j(z),\\
\frac{\partial^2 u_r}{\partial r\partial z} &= \sum_{i=0}^{n}\sum_{j=0}^{m} a_{ij}\frac{\partial T_i(r)}{\partial r}\frac{\partial T_j(z)}{\partial z},\\
\frac{\partial u_z}{\partial z} &= \sum_{i=0}^{n}\sum_{j=0}^{m} b_{ij}T_i(r)\frac{\partial T_i(z)}{\partial z},\\
\frac{\partial^2 u_z}{\partial z^2} &= \sum_{i=0}^{n}\sum_{j=0}^{m} b_{ij}T_i(r)\frac{\partial^2 T_j(z)}{\partial z^2},\\
\frac{\partial^2 u_z}{\partial r\partial z} &= \sum_{i=0}^{n}\sum_{j=0}^{m} b_{ij}\frac{\partial T_i(r)}{\partial r}\frac{\partial T_j(z)}{\partial z},\\
\frac{\partial u_z}{\partial r} &= \sum_{i=0}^{n}\sum_{j=0}^{m} b_{ij}\frac{\partial T_i(r)}{\partial r}T_j(z),\\
\frac{\partial^2 u_z}{\partial r^2} &= \sum_{i=0}^{n}\sum_{j=0}^{m} b_{ij}\frac{\partial^2 T_i(r)}{\partial r^2}T_j(z),\\
\frac{\partial^2 u_r}{\partial t^2} &= \sum_{i=0}^{n}\sum_{j=0}^{m} \ddot{a}_{ij}T_i(r)T_j(z),\\
\frac{\partial^2 u_z}{\partial t^2} &= \sum_{i=0}^{n}\sum_{j=0}^{m} \ddot{b}_{ij}T_i(r)T_j(z),
\end{aligned} \tag{33}$$

By substituting Eq. (33) in Eq. (31), a set of equations same as Eq. (26) are generated. After solving a set of equations, the unknown coefficients a_{ij} and b_{ij} are obtained.

In both cases of the homogeneous, plane strain and FGM axisymmetric cylinder by having the displacement field, the strain and stress fields can be achieved using strain-displacement and Hooke's law.

The results of static analysis for homogeneous and FGM cases are presented in Tables 4 and 5, and compared with the results obtained in a work by Tutuncu and Temel [39].

In the following part, free vibration analysis for the square and rectangular domain is done. In this example, the boundary condition (B. C.) is expressed by the symbol of ABCD that A, B, C, and D are the types of boundary conditions at plane $x = x_1$, $x = x_2$, $y = y_1$, and $y = y_2$, respectively. It should be mentioned that the symbols C and F are referred to as clamped and free boundary conditions, respectively. Tables 6 and 7 show the first five natural frequencies for a square and rectangular plate with various boundary conditions, the square plate has dimensions of $L_x = L_y = 2$ and the rectangular plate has dimensions of $L_x = 1$, $L_y = 2$. Both plates are in plane stress conditions. Furthermore, the material properties for these cases are as follows:

$$E = 207\text{GPa} \quad \nu = 0.3 \qquad \rho = 7800\text{kg/m}^3$$

From Tables 6 and 7, it is obvious that the obtained results are very close to those obtained by ANSYS, which shows the accuracy of Chebyshev spectral methods for free vibration analysis.

Finally, to show the capability of the Chebyshev collocation method for dynamic analysis, transient analysis of a cylinder was performed and compared with the results obtained with the Meshless method by Alihemmati et al. [40]. The time-dependent set of obtained differential equations in this example was solved by the Newmark method. Fig. 1. Shows the time history of radial displacement at the midpoint of cylinder obtained by the Chebyshev collocation method and also the Meshless method presented in Ref. [40].

Table 4
Stresses in radial direction for homogeneous axisymmetric cylinder.

r	σ_r (GPa)		σ_θ (GPa)	
	Ref. [39]	Present method	Ref. [39]	Present method
1	−1.0000	−1.0000	1.6666	1.6667
1.1	−0.7685	−0.7686	1.4352	1.4353
1.2	−0.5925	−0.5926	1.2592	1.2593
1.3	−0.4556	−0.4556	1.1222	1.1223
1.4	−0.3469	−0.3469	1.0136	1.0136
1.5	−0.2592	−0.2593	0.9259	0.9259
1.6	−0.1874	−0.1875	0.8541	0.8542
1.7	−0.1280	−0.1280	0.7946	0.7947
1.8	−0.0781	−0.0782	0.7448	0.7449
1.9	−0.0360	−0.0360	0.7026	0.7027
2	0.0000	0.0000	0.6666	0.6667

Table 5
Stresses in radial direction for FGM axisymmetric cylinder.

r	σ_r (GPa)		σ_θ (GPa)	
	Ref. [39]	Present method	Ref. [39]	Present method
1	−1.0000	−1.0000	1.2025	1.2026
1.1	−0.8031	−0.8032	1.1310	1.1311
1.2	−0.6443	−0.6444	1.0765	1.0765
1.3	−0.5136	−0.5137	1.0342	1.0342
1.4	−0.4043	−0.4043	1.0011	1.0012
1.5	−0.3115	−0.3115	0.9751	0.9751
1.6	−0.2317	−0.2318	0.9545	0.9545
1.7	−0.1625	−0.1625	0.9381	0.9382
1.8	−0.1017	−0.1017	0.9253	0.9253
1.9	−0.0479	−0.0480	0.9151	0.9152
2	0.0000	0.0000	0.9073	0.9073

Table 6
First five natural frequencies (Hz) of a homogeneous square domain

B.C.		ω_1	ω_2	ω_3	ω_4	ω_5
CFFF	Present method	269.8	647.5	726.4	1154.4	1244.9
	ANSYS	269.8	647.5	726.4	1154.4	1245.1
CCFF	Present method	727.6	1297.2	1341.0	1440.1	1609.3
	ANSYS	727.85	1296.8	1341.0	1439.9	1608.6
CCCC	Present method	1527.8	1527.8	1820.0	2228.5	2517.7
	ANSYS	1527.8	1527.8	1820.0	2228.5	2517.7

Table 7
First five frequencies (Hz) of a homogeneous rectangular domain.

B.C.		ω_1	ω_2	ω_3	ω_4	ω_5
CFFF	Present method	269.8	647.5	726.4	1154.4	1244.9
	ANSYS	269.8	647.5	726.4	1154.4	1245.1
CCFF	Present method	727.6	1296.8	1341.0	1439.9	1608.6
	ANSYS	727.85	1297.2	1341.0	1440.1	1609.3
CCCC	Present method	1527.8	1527.8	1820.0	2228.5	2517.7
	ANSYS	1527.8	1527.8	1820.0	2228.5	2517.7

As can be observed from Fig. 2, there is a very close agreement between two results that confirm the ability of the Chebyshev collocation method for dynamic structural analysis.

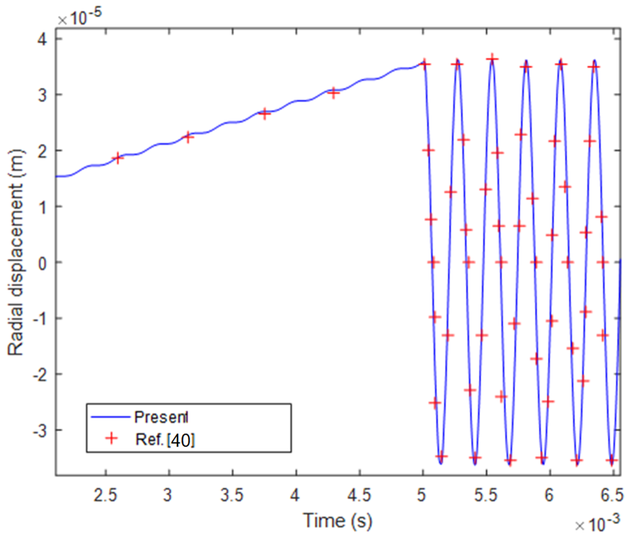

Fig. 2. Time history of radial displacement at mid-point of cylinder.

3.4. Case study 4: Axially Functionally Graded Nanobeam with Non-classical Elasticity Theories

In this case, free vibration analysis of axially functionally graded nanobeam with radius varies along the length was considered and solved with the presented method. Both stress and strain gradient theories were used. The governed equation is a six order differential equation with variable coefficients that were derived in a work by Zeighampour and Tadi Beni [41]. The governed equation is as follows:

$$-k(x)\frac{\partial^6 w}{\partial x^6} - 3\left(\frac{\partial k(x)}{\partial x}\right)\frac{\partial^5 w}{\partial x^5} + \left(s(x) - 3\frac{\partial^2 k(x)}{\partial x^2}\right)\frac{\partial^4 w}{\partial x^4} + \left(2\frac{\partial s(x)}{\partial x} - \frac{\partial^3 k(x)}{\partial x^3}\right)\frac{\partial^3 w}{\partial x^3} + \frac{\partial^2 s(x)}{\partial x^2}\frac{\partial^2 w}{\partial x^2} + m\frac{\partial^2 w}{\partial t^2} = 0 \tag{34}$$

And the boundary conditions for the clamped-clamped supports case are as below:

$$\begin{aligned} w\Big|_{x=0,L} &= 0 \\ \left(k(x)\frac{\partial w}{\partial x}\right)|_{x=0,L} &= 0 \\ \left(k(x)\frac{\partial^3 w}{\partial x^3}\right)|_{x=0,L} &= 0 \end{aligned} \tag{35}$$

This is an example that the numbers of collocation points are not equal to the number of the used Chebyshev polynomials, because at each endpoint (boundary points) there are three equations, so the total number of collocation points should be equal to the number of equations. For this example, the nano beam deflection is approximated by Chebyshev polynomials as:

$$w = \sum_{i=0}^{n} a_i T_i \tag{36}$$

And the derivates of Eq. (36) are computed as follows:

$$\begin{aligned} \frac{\partial^2 w}{\partial x^2} &= \sum_{i=0}^{n} a_i \frac{\partial^2 T_i}{\partial x^2} \\ \frac{\partial^3 w}{\partial x^3} &= \sum_{i=0}^{n} a_i \frac{\partial^3 T_i}{\partial x^3} \\ \frac{\partial^4 w}{\partial x^4} &= \sum_{i=0}^{n} a_i \frac{\partial^4 T_i}{\partial x^4} \\ \frac{\partial^5 w}{\partial x^5} &= \sum_{i=0}^{n} a_i \frac{\partial^2 T_i}{\partial x^5} \\ \frac{\partial^6 w}{\partial x^6} &= \sum_{i=0}^{n} a_i \frac{\partial^2 T_i}{\partial x^6} \\ \frac{\partial^2 w}{\partial t^2} &= \sum_{i=0}^{n} \ddot{a}_i T_i \end{aligned} \tag{37}$$

By substituting Eqs. (36) and (37) into Eqs. (34) and (35) a system of equations like Eq. (28) is generated that the natural frequencies of nanobeam can be derived from it.

The variation of mechanical properties along the length of nanobeam is according to the Ref. [41] and E_R and E_L show Young's modulus in the right and left direction of nanobeam.

Tables 8, 9, and 10 show the natural frequency parameters of nanobeam under various power low exponent (PE) and various mechanical properties. In Table 8 the results are based on classical theory, table 9 shows stress theory and Table 10 is based on strain gradient theory.

Tables 8, 9, and 10 show that there is a very good agreement between the present results and those obtained by the Differential Quadrature Method (DQM) [41]. Moreover, these results show that the Chebyshev collocation method can successfully be applied to more complex equations of structural mechanics derived based on nonclassical theory and also containing FG material properties.

Table 8
Fundamental frequency parameter for nanobeam based on classic theory.

E_L/E_R	PE	0.5	1	2
0.5	Present method	1.0082	0.9576	0.9128
0.5	Ref. [41]	1.0035	0.9553	0.9106
1	Present method	1.1217	1.1217	1.1217
1	Ref. [41]	1.1190	1.1190	1.1190
2	Present method	1.2915	1.3542	1.4117
2	Ref. [41]	1.2893	1.3510	1.4088

Table 9
Fundamental frequency parameter for nanobeam based on couple stress theory.

E_L/E_R	PE	0.5	1	2
0.5	Present method	1.1930	1.1330	1.0800
0.5	Ref. [41]	1.1874	1.1303	1.0775
1	Present method	1.3272	1.3272	1.3272
1	Ref. [41]	1.3241	1.3241	1.3241
2	Present method	1.5281	1.6023	1.6704
2	Ref. [41]	1.5255	1.5986	1.6669

Table 10
Fundamental frequency parameter for nanobeam based on strain gradient theory.

E_L/E_R	PE	0.5	1	2
0.5	Present method	1.6028	1.5235	1.4500
0.5	Ref. [41]	1.5952	1.5233	1.4498
1	Present method	1.7832	1.7832	1.7832
1	Ref. [41]	1.7829	1.7829	1.7829
2	Present method	2.0535	2.1546	2.2502
2	Ref. [41]	2.0591	2.1543	2.2499

4. Conclusions

Chebyshev collocation method was applied to static, free vibration, and dynamic analysis of some structural mechanics. The more complicated cases of equations of motion such as the case with FG material properties and a nanobeam with non-classical elasticity theory were solved by this method. It was concluded that the presented method has very good accuracy, stability, and efficiency in static, free vibration, and dynamic analysis of solid structures. Additionally, the same conclusion was observed for the heat conduction analysis of solids. Another important conclusion was that no instability of solution is observed in the present method in spite of the other collocation methods that are generally inaccurate and unstable in many situations especially for problems with second kind boundary conditions.

As a disadvantage of the method, it should be mentioned that in two-dimensional cases, this method is applicable to domains that can be mapped into square domains. For irregular domains that cannot be mapped into a square domain, these domains should be decomposed into some subdomains then each of them can be mapped separately into a square domain and apply the Chebyshev collocation method to each of them.

References

[1] C. Canuto, M.Y. Hussaini, A. Quarteroni, T.A. Zang, Spectral Methods: Evolution to Complex Geometries and Applications to Fluid Dynamics, Springer-Verlag, Berlin Heidelberg, (2007).

[2] M. Krawczuk, M. Palacz, W. Ostachowicz, The dynamic analysis of a cracked Timoshenko beam by the spectral element method, J. Sound Vib., 264(5) (2003) 1139-1153.

[3] Dale R. Durran, Numerical Methods for Fluid Dynamics With Applications to Geophysics, Second Edition, Springer-Verlag, New York, Seattle, (2010).

[4] P. Grandcl´ement, Introduction to spectral methods, EAS Publications Series, (2006) DOI: 10.1051/eas:2006112.

[5] A. Pinelli, A. Vacca, Chebyshev collocation method and multidomain decomposition for the incompress-

ible navier-stokes equations, Int. J. Numer. Methods Fluids, 18(8) (1994) 781-799.

[6] X. Zhang, X.H. Liu, K.Z. Song, M.W. Lu, Least-squares collocation meshless method, Int. J. Numer. Methods Eng., 51 (2001) 1089-1100.

[7] X. Liu, G.R. Liu, K. Tai, K.Y. Lam, Radial basis point interpolation collocation method for 2-dsolid problem, Advances in Meshfree and X-FEM Methods, (2002) 35-40. DOI: 10.1142/9789812778611_0008.

[8] S.H. Lee and Y.C. Yoon, Meshfree point collocation method for elasticityand crack problems, Int. J. Numer. Methods Eng., 61(1) (2004) 22-48.

[9] O. Civalek, O. Kiracioglu, Free vibration analysis of Timoshenko beams by DSC method, Int. J. Numer. Meth. Biomed. Eng., 26(12) (2010) 1890-1898.

[10] O. Civalek, A. Yavas, Large Deflection Static Analysis of Rectangular Plates On Two Parameter Elastic Foundations, Int. J. Sci. Tech., 1(1) (2006) 43-50.

[11] K. Mercan, C. Demir, O. Civalek, Vibration analysis of FG cylindrical shells with power-law index using discrete singular convolution technique, Curved and Layer. Struct., 3 (1) (2016) 82-90.

[12] H. Nourmohammadi, B. Behjat, Static analysis of functionally graded piezoelectric plates under electro-thermo-mechanical loading using a meshfree method based on RPIM, J. Stress Anal., 4(2) (2020) 93-106.

[13] M. Sheikhi Azqandia, M. Hassanzadeh, M. Arjmand, Calculation of design shape sensitivity in solid mechanics through a novel hybrid method using CVM and DSM, J. Stress Anal., 5(1) (2020) 11-20.

[14] D. Zhou, Y.K. Cheung, F.T.K. Au, S.H. Lo, Three-dimensional vibration analysis of thick rectangular plates using Chebyshev polynomial and Ritz method, Int. J. Solids Struct., 39(26) (2002) 6339-6353.

[15] I. Celik, Solution of magnetohydrodynamic flow in a rectangular ductby Chebyshev collocation method, Int. J. Numer. Methods Fluids, 66(10) (2011) 1325-1340.

[16] J.M. Carcione, A 2D Chebyshev differential operator for the elastic wave equation, Comput. Methods Appl. Mech. Eng., 130(1-2) (1996) 33-45.

[17] U. Ehrenstein, R. Peyret, A chebyshev collocation method for the navier-stokes equations with application todouble-diffusive convection, Int. J. Numer. Methods Fluids, 9(4) (1989) 427-452.

[18] L.Z. Wu, S.R. Zhu, J. Peng, Application of the Chebyshev spectral method to the simulation of groundwater flow and rainfall-induced landslides, Appl. Math. Modell., 80 (2020) 408-425.

[19] Y. Huang, Y. Zhao, T. Wang, H. Tian, A new Chebyshev spectral approach for vibration of in-plane functionally graded Mindlin plates with variable thickness, Appl. Math. Modell., 74 (2019) 21-42.

[20] R. Rani, R. Lal, Free vibrations of composite sandwich plates by Chebyshev collocation technique, Composites Part B, 165 (2019) 442-455.

[21] N.K. Guru, S.K. Jain, Free axisymmetric vibrations of composite annular sandwich plates by higher-order theory using Chebyshev collocation technique, Thin Walled Struct., 155 (2020) 106823.

[22] J. Alihemmati, Y. Tadi Beni, Y. Kiani, Application of Chebyshev collocation method to unified generalized thermoelasticity of a finite domain, J. Therm. Stresses, 44(5) (2020) 547-565.

[23] J. Alihemmati, Y. Tadi Beni, Y. Kiani, LS-based and GL-based thermoelasticity in two dimensional bounded media: A Chebyshev collocation analysis, J. Therm. Stresses, (2021), DOI: 10.1080/01495739.2021.1922112.

[24] C.H. Lin, M.H.R. Jen, Analysis of a laminated anisotropic plate by Chebyshev collocation method, Compos. B. Eng., 36(2) (2005) 155-169.

[25] S. Gumgum, E. Kurul, N. Baykus Savasaneril, Chebyshev collocation method for the two-dimensional heat equation, Communication in Mathematical Modeling and Applications, 3(2) (2018) 1-8.

[26] Ö. Civalek, C. Demir, A simple mathematical model of microtubules surrounded by an elastic matrix by nonlocal finite element method, Appl. Math. Comput., 289 (2016) 335-352.

[27] M. Mohtashami, Y. Tadi Beni, Size-dependent buckling and vibrations of piezoelectric nanobeam with finite element method, Iran J. Sci. Technol. Trans. Civ. Eng., 43 (2019) 563-576.

[28] M. Gürses, B. Akgöz, Ö. Civalek, Mathematical modeling of vibration problem of nano-sized annular sector plates using the nonlocal continuum theory via eight-node discrete singular convolution transformation, Appl. Math. Comput., 219(6) (2012) 3226-3240.

[29] F. Karami, A. Ahmadi Nadooshan, Y. Tadi Beni, Analytical solution of Newtonian nanofluid flow in a tapered artery based on a consistent couple stress theory, Heat Mass Transf., 56 (2020) 459-476.

[30] I. Soleimani, Y. Tadi Beni, M. Botshekanan Dehkordi, Size-dependent two-node axisymmetric shell element for buckling analysis with couple stress theory, Proc. Inst. Mech. Eng., Part C: J. Mech. Eng. Sci., 233(13) (2019) 4729-4741.

[31] I. Soleimani, Y. Tadi Beni, Vibration analysis of nanotubes based on two-node size dependent axisymmetric shell element, Arch. Civ. Mech. Eng., 18(4) (2018) 1345-1358.

[32] C. Demir, O. Civalek, On the analysis of microbeams, Int. J. Eng. Sci., 121 (2017) 14-33.

[33] Y. Tadi Beni, Size-dependent electromechanical bending, buckling, and free vibration analysis of functionally graded piezoelectric nanobeams, J. Intell. Mater. Syst. Struct., 27(16) (2016) 2199-2215.

[34] I. Soleimani, Y. Tadi Beni, F. Mehralian, A new size-dependent cylindrical shell element based on modified couple stress theory, Adv. Appl. Math. Mech., 10(4) (2018) 819-844.

[35] Sh. Dastjerdi, Y. Tadi Beni, A novel approach for nonlinear bending response of macro- and nanoplates with irregular variable thickness under nonuniform loading in thermal environment, Mech. Based Des. Struct. Mach., 47(4) (2019) 453-478.

[36] J. Alihemmati, Y. Tadi Beni, Developing three-dimensional mesh-free Galerkin method for structural analysis of general polygonal geometries, Eng. Comput., 36 (2020) 1059-1068.

[37] V.S. Arpaci, Conduction heattransfer, Addison-Wesley, Massachusetts, (1966).

[38] W. Michael Lai, D. Rubln, E. Krempl, Introduction to Continuum Mechanics, Third Edition, Butterworth-Heinemann, Woburn, (1994).

[39] N. Tutuncu, B. Temel, A novel approach to stress analysis of pressurized FGM cylinders, disks and spheres, Compos. Struct., 91(3) (2009) 385-390.

[40] J. Alihemmati, M. Foroutan, A. Soltanimaleki, Dynamic analysis of functionally graded fiber reinforced cylinders under an impact load by a three dimensional mesh free approach, J. Dyn. Behav. Mater., 3 (2017) 391-406.

[41] H. Zeighampour, Y. Tadi Beni, Free vibration analysis of axially functionally graded nanobeam with radius varies along the length based on strain gradient theory, Appl. Math. Model., 39(18) (2015) 5354-5369.